

Analysis of a Linear Multihop Underwater Acoustic Network

Wenyi Zhang, *Member, IEEE*, Milica Stojanovic, *Fellow, IEEE*, and Urbashi Mitra, *Fellow, IEEE*

Abstract—In this paper, a multihop underwater acoustic line network, which consists of a series of equal-distance hops connected by relay transceivers in a tandem, is considered. Messages are originated as coded packets from a source node at one end, relayed sequentially hop by hop (decoded and re-encoded), and finally received by a destination node at the other end of the network. Several key characteristics of underwater acoustic channels, namely, frequency-dependent signal attenuation and noise, interhop interference, half-duplex modem constraint, and large propagation delay, are taken into account in the analysis. Simple transmission protocols with spatial reuse and periodic transmit/receive schedule are considered. Performance bounds and scheduling design are developed to satisfy the half-duplex constraint on relay transceivers in the presence of long propagation delay. To efficiently cope with frequency-dependent channel characteristic and interhop interference, the power spectral density (PSD) of the signaling is analytically optimized in a way analogous to water filling. Furthermore, the problem of determining the minimum number of hops to support a prespecified rate and reliability with and without a maximum coded packet length constraint is examined. Finally, numerical results are presented to illustrate the analysis.

Index Terms—Interference, multihop network, propagation delay, reliability, scheduling, spectral shaping, underwater acoustic communication.

I. INTRODUCTION

IN underwater acoustic communication systems, both bandwidth and power are severely limited due to the attenuation that depends exponentially, rather than polynomially, on propagation distance, and also, on frequency. As a consequence, multihop transmission, in which a longer distance is divided into multiple shorter hops, offers more favorable bandwidth

and path loss conditions, and appears as an attractive solution for providing high-rate services for next-generation underwater acoustic communications [2], [3]. The aim of this paper is to give a preliminary analysis of multihop networking in underwater acoustic environments.

To gain insights from the analysis, we consider a simple network model, in which several hops, each of identical distance, are connected in a tandem, and information-bearing data packets originating from a source node at one end of the tandem are forwarded hop by hop to a destination node at the other end of the tandem. Specifically, a relay acoustic transceiver node is employed in between consecutive hops. The relay nodes receive the incoming packets, decode them, and retransmit them to the next hop, until the final destination is reached. Such a model, though simple, captures the essential elements of multihop networking, and its analysis reveals interesting features, as will be shown in the paper.

For multihop networks equipped with full-duplex relay transceivers, and operating over wireline type of links without interference among hops, the network capacity is easily shown (by the standard cut-set bound [4, Th. 14.10.1] in information theory) to be the minimum link capacity among hops, and the capacity is straightforwardly achieved by implementing good error-control coding for each hop. As a consequence, a significant portion of the literature on the information-theoretic aspects of multihop networking has focused on the network capacity for noncoding relays; see, e.g., [5]–[7] and references therein. The reliability-delay tradeoff in such type of multihop networks was addressed in [8].

However, the situation dramatically changes for multihop networks with half-duplex relay transceivers and with interference among hops. Such a situation arises in wireless radio links, or underwater acoustic links, which we consider in this paper. The interference among hops is due to the broadcasting nature of wireless (either radio or underwater acoustic) communications, and it fundamentally changes the network model from a simple multihop tandem to a general relay network, for which the capacity problem is extremely difficult and still open [9]. It is not the aim of this paper to solve the general information-theoretic problem, but instead, we consider several suboptimal transmission protocols which explicitly consider features of underwater acoustic channels, and analyze their performance. The transmission protocols considered are of a flavor similar to that in [10], in which for each time slot, a certain subset of nodes simultaneously transmit in the same frequency band with the same average power constraint. Such a form of spatial reuse increases the spectrum utilization efficiency, while at the potential risk of introducing excess interference among receiving nodes. Hence, there exists an optimal reuse factor, which we identify

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in the paper. At the same time, due to the frequency-dependent signal-to-noise ratio (SNR) characteristic of underwater acoustic links, as well as the existence of interference, the optimal spectral shape of the signaling scheme can significantly deviate from both flat and classical water-filling solutions [9]. In the paper, we solve the spectral shaping problem in the presence of interference.

We further consider the problem of determining the minimum number of hops between two fixed end nodes (source and destination), for supporting a prespecified rate and reliability, which is measured in terms of packet error probability. Such a problem arises from the fact that node deployment is extremely costly for underwater environments, and it is wasteful if more nodes than necessary are used. The problem of determining the optimal number of hops for a wireless relay link has been addressed in [10] under idealized conditions; here, we focus on more realistic conditions that take the practical issues of coding and delay into account. We find conditions for determining the minimum number of hops, under either the ideal situation of capacity-achieving coding with sufficiently long packets, or the more realistic situation of reliability-rate tradeoff with finite packet lengths. We note that for underwater acoustic communication systems the largest delay contribution is from propagation at the low speed of sound, thus decreasing the end-to-end coding delay is not our objective in the paper.

Multihop networking for underwater acoustic systems has attracted a heightened interest recently; see [11]–[16] and references therein. However, we note that most of the existing works are focused on particular transceivers' design and analysis, and not the information-theoretic performance limits, as we will address in this paper.

The remainder of the paper is organized as follows. In Section II, we describe the multihop network model. We develop a performance bound and design transmission scheduling with spatial reuse in Section III. In Section IV, we analyze the impact of spatial reuse and signaling power spectral density (PSD) on achievable rates of multihop networks, and provide a general method of evaluating the network protocol capacity. We address the problem of determining the minimal number of hops for supporting prespecified rate and reliability requirements in Section V. In Section VI, we present numerical results, which aim to illustrate the analysis of the previous sections, and to provide insights into the typical behavior of multihop underwater acoustic networks. Finally, in Section VII, we conclude the paper with a discussion.

II. MULTIHOP NETWORK MODEL

In this section, we describe the multihop network model, as illustrated in Fig. 1. Nodes are denoted by \mathcal{N}_k , $k = 0, 1, \dots, K$, among which \mathcal{N}_0 is the source, \mathcal{N}_K is the destination, and the remaining nodes are relays located between \mathcal{N}_0 and \mathcal{N}_K . For analytical simplicity, we assume that all the nodes are located along a straight line, and that every two adjacent nodes are separated by the same distance d . It is possible to modify this transmission scheme to treat multihop line networks with nonuniform hop distances; however, the generalization does not provide additional intuition.

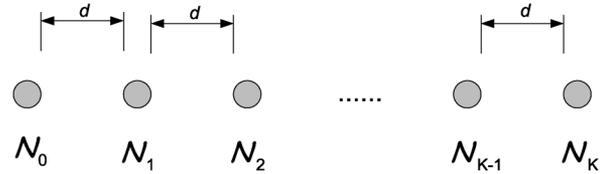


Fig. 1. Illustration of a multihop network model. All nodes are located along a straight line, and every two adjacent nodes are distance d apart. Messages are transmitted from the source node \mathcal{N}_0 , sequentially along $\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_{K-1}$, to the destination node \mathcal{N}_K .

For the moment, the number of hops K and the hop distance d are treated as separate parameters. In Section V, we will further fix the total source–destination distance such that d is inversely proportional to K .

A. Frequency-Domain Representation of Link SNR Model

As a signal propagates and is received by a node, its energy dissipates and the signal is contaminated by multipath and noise. Let us start with modeling the frequency and distance-dependent attenuation for a single path. For wireless radio links, the frequency dependency of the path loss is typically negligible over bandwidths used in most applications, and it is common practice to approximate the attenuation as $A(d) \propto d^\alpha$, where d is the transmission distance and α is a constant decay factor [17]. However, for underwater acoustic links, both the link distance and the signaling frequency have significant impact on the attenuation, which we denote by $A(d, f)$. Consequently, for a transmitted signal $s(t)$, the signal received over a single propagation path without noise can be described as the convolution

$$r(t) = a(d, t) * s(t) \quad (1)$$

where $a(d, t) = \mathcal{F}^{-1}[1/\sqrt{A(d, f)}]$ denotes the inverse Fourier transform of $1/\sqrt{A(d, f)}$, i.e., the impulse response due to frequency-dependent attenuation. The attenuation, or the path loss that occurs over a distance d kilometers for a narrowband signal of carrier frequency f kilohertz, obeys [3]

$$A(d, f) \propto d^\kappa a(f)^d \quad (2)$$

where κ is the spreading factor, which we take as $\kappa = 1.5$ (practical spreading), and the frequency-dependent absorption coefficient $a(f)$ is an increasing function of frequency, which can be obtained using empirical formulas [3].

Assuming the absence of site-specific noise, the receiver is affected only by the ambient noise $n(t)$, which we assume to be a zero-mean circularly symmetric complex Gaussian process, and whose PSD $N(f)$ is also obtained empirically [3].

Now let us turn to modeling the effect of multipath. The effect of multipath can be significant due to sound reflection from the surface, bottom, and other objects. We consider a shallow-water scenario, so that the effect of sound refraction is ignored. The general treatment of multipath in the underwater acoustic channel is given in [18]; here, we focus on multicarrier signaling, viewing the signal as a collection of multiple narrowband signals, each affected by attenuation $A(d, f)$ and noise of PSD $N(f)$ at a given frequency f . We will assume that channel

variation in time is slow enough that it can be tracked by the receiver.

Let us assume that there is a number of discrete paths, each of which has a path distance denoted by d_l , a cumulative reflection coefficient Γ_l , and a propagation delay $\tau_l = d_l/c$ where c denotes the speed of sound underwater, for $l = 0, 1, \dots$. The frequency-domain channel response $H(f)$ is given by

$$H(d, f) = \sum_{l \geq 0} \frac{\Gamma_l}{\sqrt{A(d_l, f)}} e^{-j2\pi f \tau_l} \quad (3)$$

where d denotes the horizontal distance between the transmitter and the receiver. Without loss of generality, we assume that all the nodes are at the same depth, and that they are near the bottom. Associated with the channel transfer function is a frequency-dependent SNR characteristic, which we define as

$$\rho(d, f) = \frac{|H(d, f)|^2}{N(f)}. \quad (4)$$

We will use this characteristic in Section IV.

B. Slotted Packet Transmission Model

Propagation delay in an underwater acoustic channel is typically long, and it needs to be carefully treated in the design of transmission protocols. The speed of sound underwater is approximately $c = 1.5$ km/s, and the hop delay $\tau = d/c$ can be as long as a fraction of a second or even several seconds, much greater than the packet duration.

We adopt a slotted model for packet transmission. Each packet corresponds to one time slot of duration T . The value of T is assumed to be substantially greater than the multipath delay spread, and a guard time inserted at the end of each packet to eliminate interpacket interference incurs a negligible rate loss. Furthermore, for analytical convenience, we design transmission protocols such that τ/T is a positive integer, denoted by D . With such a design, it is possible to consider synchronous transmission protocols in which all the nodes transmit, receive, or remain idle on a common regular time grid, as we will discuss in detail in Section III. For the moment, we note that in each time slot, a node is in one of three modes: transmitting a packet, receiving a packet, or idle, denoted by **T**, **R**, and **I**, respectively. Due to the half-duplex constraint of the currently available acoustic modems, a node cannot simultaneously be in multiple modes.

III. TRANSMISSION PROTOCOL WITH SPATIAL REUSE

The half-duplex constraint requires scheduled transmissions to avoid activating the transmit (**T**) and receive (**R**) modes of a node in the same time slot. There are various specific ways of scheduling in the presence of a long propagation delay; see e.g., [1] and [19] for a few *ad hoc* designs. In this section, we present a systematic approach that provides a more general scheduling solution, whose performance exceeds our prior work as presented in [1] and [19]. To improve the spectral efficiency, the idea of *spatial reuse* is critical, in which for each time slot, a certain subset of nodes transmit while another subset of nodes receive.

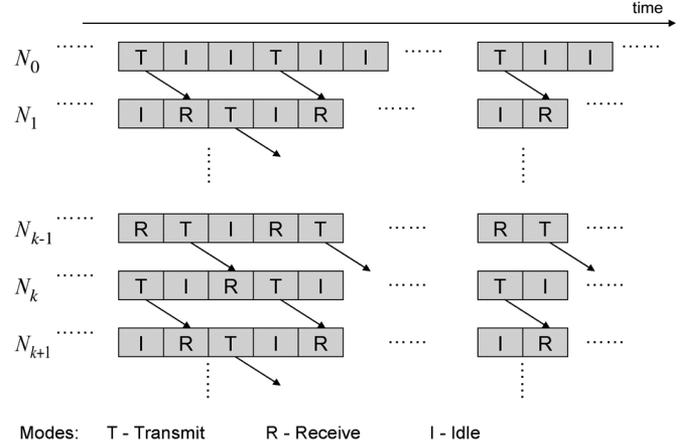


Fig. 2. Transmission schedule is represented by a time-space chart.

A transmission protocol has a spatial reuse factor Q if for each node (except the destination which only receives), on average, a fraction $1/Q$ of its time slots are used for transmission. For example, the following periodic scheduling sequence has a spatial reuse factor $Q = 3$:

$$\dots \underbrace{\mathbf{T, I, R}}_{Q=3}, \underbrace{\mathbf{T, I, R}}_{Q=3}, \underbrace{\mathbf{T, I, R}}_{Q=3}, \dots$$

For simplicity, we require that Q be an integer. Due to the half-duplex constraint, the minimum value of Q is two.

We do not consider sophisticated techniques for interference cancellation in this paper, and simply treat the interference as noise. The interference originates from nodes upstream and downstream from the receiver, not counting the intended transmitter, i.e., the first upstream neighbor. A transmission protocol should thus minimize at each node the aggregate interference from all of its interfering nodes. This problem does not appear amenable to analysis. Therefore, we take an alternative approach which maximizes the distance between each node and its nearest interfering node. Since the hops are of equal distance, maximizing this distance is equivalent to maximizing the number of hops that an interfering signal traverses before reaching the receiver. The rationale behind this approach is that the acoustic attenuation rapidly increases with distance, and, hence, the nearest interfering node contributes the most to the total interference.

A. An Upper Bound on the Distance of the Nearest Interfering Node

We first establish an upper bound on the maximum number of hops between a node and its nearest interfering node.

Proposition 1: For any transmission protocol with a spatial reuse factor Q (Q is assumed to be smaller than the number of nodes in the network), there always exists at least one node, such that the maximum number of hops between this node and its nearest interfering node is at most $Q + 1$.

Proof: See part A of the Appendix.

Remark: In light of Proposition 1, when the spatial reuse factor Q is fixed, we cannot arbitrarily decrease the amount of interference.

clear distinction, we call the derived achievable rate the *protocol* capacity.

Let us consider the K -hop network model of Section II. For simplicity, we impose an identical average power constraint of P on all the transmit nodes. Although unequal power allocation among nodes may lead to certain performance gain (but it is not clear that it does), the resulting optimization problem would be nontrivial even for simple wireless radio model without frequency-dependent attenuation [10]. Thus, we focus on optimizing a single PSD of the transmitted signal so as to maximize the link capacity in the presence of interference. Note also that using the same transmitter filter for all the nodes greatly eases implementation and deployment.

To facilitate the analysis, we let the number of hops K become infinitely large. This is a worst case scenario in that the interference power is maximized, and the corresponding protocol capacity hence serves as a lower bound on the performance of networks with finite size. However, our numerical investigation shows that practically all of the interference is contributed by the nearest interfering node $Q + 1$ hops away, and, hence, the infinite-node approximation in fact has a satisfactory accuracy even for networks of a small size.

By inspecting the transmission schedule of Section III-B, we notice that for a receive node, the interference comes only from earlier transmissions of upstream nodes at distances $(Q + 1)d, (2Q + 1)d, \dots, (iQ + 1)d, \dots$. Considering an infinitesimally narrow bandwidth centered around a carrier frequency f , and assuming that all the nodes transmit the signal of PSD $S(f)$, we have that the total interference PSD is

$$I(f) = \sum_{i=1}^{\infty} |H((iQ + 1)d, f)|^2 S(f). \quad (5)$$

The interference-to-noise PSD ratio can then be expressed as

$$I(f)/N(f) = S(f)\rho_I(Q, d, f) \quad (6)$$

where

$$\rho_I(Q, d, f) = \sum_{i=1}^{\infty} \rho((iQ + 1)d, f) \quad (7)$$

with $\rho(\cdot, \cdot)$ given by (4) in Section II.

The signal-to-interference-plus-noise PSD ratio can consequently be evaluated as

$$\frac{|H(d, f)|^2 S(f)}{I(f) + N(f)} = \frac{\rho(d, f)S(f)}{1 + \rho_I(Q, d, f)S(f)}.$$

The information rate achievable with the PSD $S(f)$ is given by

$$R(d, Q, S(f)) = \frac{1}{Q} \int_{f \geq 0} \log \left(1 + \frac{\rho(d, f)S(f)}{1 + \rho_I(Q, d, f)S(f)} \right) df \quad (8)$$

where the $1/Q$ scaling factor is because each node is active only $1/Q$ of the time with spatial reuse. For a given hop distance d , we can thus optimize (8) over Q and $S(f)$ to maximize the rate $R(d, Q, S(f))$. For every fixed integer $Q \geq 2$, optimization over $S(f)$ yields (9) shown at the bottom of the page, if $\rho(d, f) \geq \lambda$; and $S_Q(f) = 0$ otherwise. The parameter $\lambda > 0$ is chosen such that

$$\int_{f \geq 0} S_Q(f) df = P \quad (10)$$

is satisfied. For proof, see part B of the Appendix. This solution is analogous to the well-known water-filling solution for colored Gaussian channels [4, ch. 10.5], and the parameter λ is the corresponding Lagrangian multiplier. Since Q is integer valued, optimization over Q can be performed using a discrete search.

To compute the optimal signal PSD, we start with a sufficiently small $\lambda > 0$, which would lead to $S(f)$ such that $\int_{f \geq 0} S(f) df > P$. We then gradually increase λ , until the condition (10) is satisfied. The corresponding λ thus yields the optimal $S_Q(f)$. As we optimize the signal PSD for every integer $Q \geq 2$, we obtain the maximum information rate for that Q

$$R(d, Q) = \max_{S(f)} R(d, Q, S(f)) = R(d, Q, S_Q(f)). \quad (11)$$

The protocol capacity can now be evaluated as

$$C(d) = \max_{Q, S(f)} R(d, Q, S(f)) = \max_Q R(d, Q). \quad (12)$$

Here we use $C(d)$ to explicitly indicate the fact that the protocol capacity of a multihop relay link depends on the hop distance d .

From the form of (12), it appears that we need to perform an exhaustive search over all integers $Q \geq 2$. In fact, it is not necessary to do so over a large range of Q 's. For a fixed Q , an immediate upper bound to $R(d, Q, S(f))$ is the capacity of one hop without interference, normalized by Q , which monotonically decreases to zero as Q increases. Therefore, as soon as the upper bound for a certain Q_1 falls below $R(d, Q_2)$ for any $Q_2 < Q_1$, we can stop searching beyond $Q = Q_1$.

V. MINIMUM NUMBER OF HOPS FOR PRESPECIFIED RATE AND RELIABILITY

The analysis of Section IV provides a general method for evaluating the information rates of multihop networks with spatial reuse. In this section, we turn to an application of that analysis. We fix the end-to-end distance between the source and the destination to d_{tot} , and consider how many hops are necessary to support a prescribed rate with a prescribed reliability, measured by an upper bound on the packet error probability.

Since packets are coded, we need to relate the reliability and the information rate, on the basis of a fixed maximum packet

$$S_Q(f) = \frac{1}{2\rho_I(Q, d, f) [\rho(d, f) + \rho_I(Q, d, f)]} \times \left\{ -[\rho(d, f) + 2\rho_I(Q, d, f)] + \sqrt{\rho^2(d, f) + (4/\lambda)\rho(d, f)\rho_I(Q, d, f) [\rho(d, f) + \rho_I(Q, d, f)]} \right\} \quad (9)$$

length. To do so, we use a general reliability function $E_C(R, T)$, which will be instrumental in the analysis of this section. For a specific channel parameterized by a generic parameter \mathcal{C} (for example, SNR in additive white Gaussian noise channels, or crossover probability in binary symmetric channels), a specific coding scheme with information rate R , coding block length T , and a specific decoding algorithm $E_C(R, T)$ is the corresponding block error probability. Generally speaking, $E_C(R, T)$ may be obtained either analytically (see, e.g., [20]), or by Monte Carlo simulations. In information theory, $E_C(R, T)$ is often interpreted as the minimum block error probability (or its upper and lower bounds) for optimal codes and optimal (maximum-likelihood) decoding algorithms (see, e.g., [21]).

For a properly designed coded system, $E_C(R, T)$ satisfies the following properties.

- 1) $E_C(R, T)$ is nonincreasing in coding block length T , i.e., longer codes lead to smaller error rates.
- 2) $E_C(R, T)$ is nondecreasing in information rate R , i.e., increased information rate leads to increased error rates.
- 3) For rates below a certain threshold (for example, the channel capacity), there exist a coding scheme and a decoding algorithm such that $E_C(R, T)$ decreases toward zero as $T \rightarrow \infty$.
- 4) For rates above the threshold in 3), $E_C(R, T)$ is always bounded away from zero.

Now let us return to the problem of determining the minimum number of hops. We consider dividing the distance d_{tot} between the source and the destination into K hops, each of which has a link distance d_{tot}/K . Meanwhile, we fix the total power as P_{tot} , and thus each node, when transmitting, is allocated an average power of $(Q/K)P_{\text{tot}}$ if the transmission protocol has a spatial reuse factor of Q (assuming $Q \leq K$). From the discussion in Section IV, we have that the protocol capacity $C(d_{\text{tot}}/K)$ is given by (12). We assume that the system design requires a pre-specified information rate R_0 , with a pre-specified end-to-end packet error probability no greater than E_0 . For many practical applications, E_0 is a number close to zero, say 10^{-4} , and the number of hops K is moderate. Therefore, the union bounding technique provides a reasonably accurate reliability constraint for each hop as E_0/K . Due to the complexity and network-layer considerations, we further assume that the coded packets have a maximum length constraint of T_{max} .

We first consider an ideal scenario, where the maximum packet length T_{max} is sufficiently large, such that all rates below $C(d_{\text{tot}}/K)$ can be achieved while satisfying the per-hop reliability constraint E_0/K . Therefore, the number of hops should satisfy

$$C\left(\frac{d_{\text{tot}}}{K}\right) \geq R_0 \quad (13)$$

and the minimum number of hops immediately follows. The inequality (13) simply asserts that the number of hops should be chosen such that the hop capacity exceeds the pre-specified information rate.

We then consider the more realistic scenario where T_{max} is finite. Using the number of hops K as a parameter, we have that it suffices to choose K such that

$$E_K(R_0, T_{\text{max}}) \leq \frac{E_0}{K}. \quad (14)$$

That is, for each hop, the coding scheme achieves an average packet error probability no greater than E_0/K , at information rate R_0 , and with packet length T_{max} . Here we use the subscript K to particularize the generic channel parameter \mathcal{C} , since for a given system model, specifying the number of hops K determines the channel response for the resulting hops.

Determining the reliability function $E_K(R, T)$ is rather tedious, and it depends upon the particular coding scheme and the decoding algorithm used. For simplicity, we evaluate $E_K(R, T)$ as the random-coding exponential error bound [21], i.e., $E_K(R, T) = \exp[-T \cdot E_r(R)]$, where $E_r(R)$ is the random-coding error exponent. The mathematical expressions used for numerical evaluation in Section VI are given in part C of the Appendix. With this treatment of the packet error rate, the minimum number of hops should have the resulting random-coding error exponent satisfy

$$E_r(R_0) \geq \frac{1}{T_{\text{max}}} \log\left(\frac{K}{E_0}\right). \quad (15)$$

VI. NUMERICAL RESULTS

In this section, we illustrate the analytical results numerically, for the underwater acoustic link model described in Section II-A. We take the link parameters as $\kappa = 1.5$ (practical spreading), $s = 0.5$ (moderate shipping activity), and $w = 0$ (calm seas). For convenience, we normalize the SNR link characteristic $\rho(d, f)$ such that $\max_f \rho(1 \text{ km}, f) = 0$ dB in the absence of multipath. To illustrate the effect of multipath, we assume that the nodes are mounted near the bottom and there are only two resolvable paths: the direct path and the path reflected from the surface. The cumulative reflection coefficient of the surface reflection is taken as -1 . The water depth is taken as 50 m throughout this section. Note that as the hop distance changes, the multipath coefficients will be changed correspondingly.

As illustrated in [18, Fig. 1], the SNR characteristic $\rho(d, f)$ degrades with distance quite severely. For each link distance, there is a particular frequency at which the received SNR is maximized, and the resulting maximal SNR decreases rapidly as the link distance increases. Furthermore, the SNR vanishes approximately in a decibel-linear fashion at high frequencies, and the decay rate grows as the link distance increases.

The interference-to-noise characteristic $\rho_I(Q, d, f)$ is plotted in Fig. 4 as a function of f , for $d = 0.5$ km, and for spatial reuse factor $Q = 2, 3$, and 4. We observe that for each Q , the $\rho_I(Q, d, f)$ curve closely follows a similar trend as $\rho(d, f)$. In fact, our numerical evaluation shows that the greatest part of $\rho_I(Q, d, f)$ is due to the nearest interfering node (at distance $(Q+1)d$ away from the receiver). Therefore, the similarity between $\rho_I(Q, d, f)$ and $\rho(d, f)$ is not entirely surprising. Such a rapid convergence in $\rho_I(Q, d, f)$ is largely due to the exponential term $a(f)^d$ in the attenuation function $A(d, f)$ in (2), which is a key difference between underwater acoustic channels and wireless radio channels, where interference coming from remote nodes decays slowly. As the reuse factor Q increases, $\rho_I(Q, d, f)$ decreases rapidly, resulting in less interference for the receiving node. However, the $1/Q$ scaling factor in the achievable rate $R(d, Q)$ also takes effect. As a consequence,

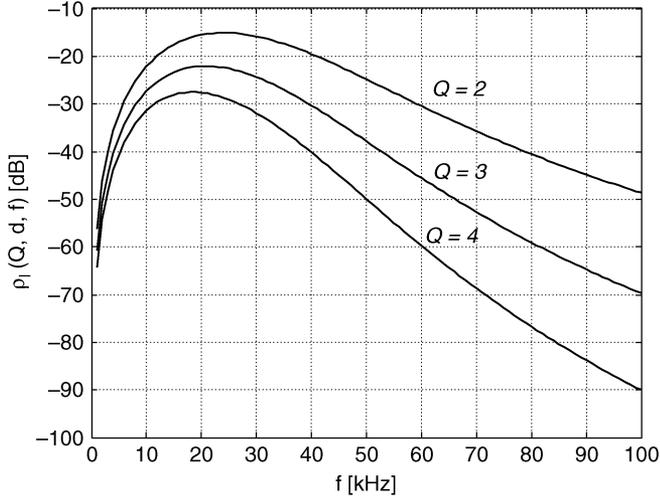


Fig. 4. Interference-to-noise ratio characteristic $\rho_I(Q, d, f)$ as a function of frequency f , for the link distance $d = 0.5$ km, and several reuse factors Q .

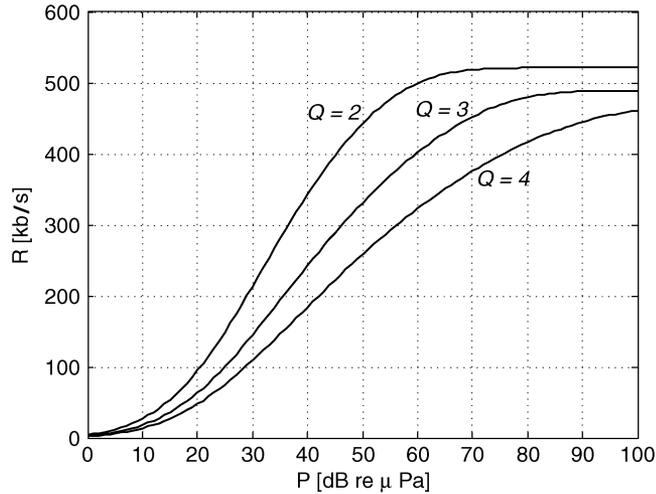


Fig. 5. Achievable information rates (optimized over the signal PSD) versus per-node power, for hop distance $d = 0.5$ km, and several reuse factors Q .

it is optimal to operate at $Q = 2$, at least in the examples considered below.

We compute the optimal signal PSD following (9), and plot achievable rates as functions of per-node power P in Figs. 5 and 6, for hop distances $d = 0.5$ and 10 km, respectively. From both figures, we observe that the optimal reuse factor Q is in fact always two, for the range of P plotted. In Fig. 5, which corresponds to a smaller hop distance, the achievable rates saturate at large P , reflecting the fact that the interhop interference becomes dominant there. In Fig. 6, which corresponds to a different distance, the saturation phenomenon has not occurred for the range of P plotted, implying that the increased hop distance also significantly reduces the amount of interhop interference.

Now we proceed to determine the minimum number of hops for supporting a prespecified rate and reliability, as defined in Section V. For the ideal scenario given by (13), we can compute the protocol capacity of a K -hop network, $C(d_{\text{tot}}/K)$, and find the minimum value of K for which (13) is satisfied. As an example, we plot in Fig. 7 the $C(d_{\text{tot}}/K)$ versus K , for $d_{\text{tot}} = 10$

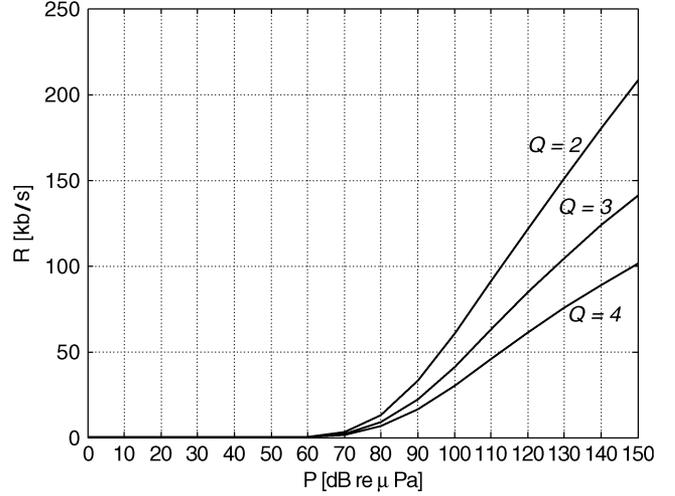


Fig. 6. Achievable information rates (optimized over the signal PSD) versus per-node power constraint, for hop distance $d = 10$ km, and several reuse factors Q .

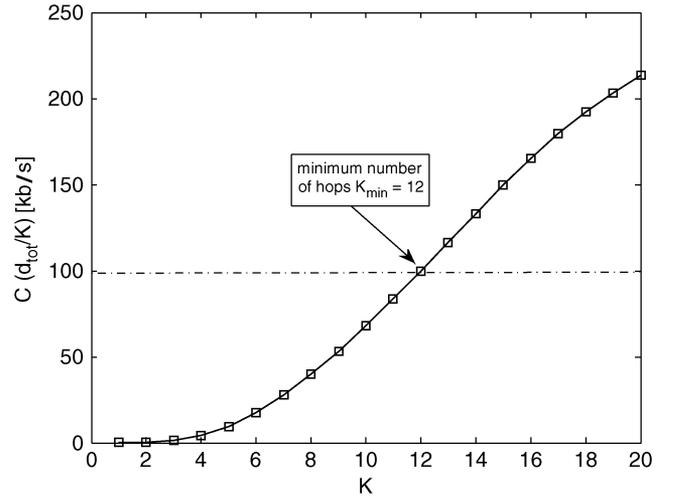


Fig. 7. Finding the minimal number of hops necessary to support a prespecified rate. The curve with squares indicates the relationship between the number of hops and the resulting protocol capacity, and the first square strictly above the prespecified rate corresponds to the minimum number of hops, which is $K_{\min} = 12$ in this plot. System parameters are $d_{\text{tot}} = 10$ km, $P_{\text{tot}} = 40$ dB re μPa (normalized), and $R_0 = 100$ kb/s.

km, $P_{\text{tot}} = 40$ dB re μPa .¹ We observe that if the prespecified information rate is $R_0 = 100$ kb/s, then the minimum number of hops should be chosen as $K_{\min} = 12$, implying that the hop distance is $d_{\text{tot}}/K_{\min} \approx 833$ m.

Next we consider a more realistic scenario, in which both rate and reliability (packet error probability) affect the minimum number of hops, as indicated by (14). In Fig. 8, two curves are plotted: the curve with squares indicates the relationship between K and $E_r(R_0)$, and the dashed-dotted curve represents $(1/T_{\text{max}}) \log(K/E_0)$. System parameters are the same as those of Fig. 7, and additionally we assume $E_0 = 10^{-4}$ and $T_{\text{max}} = 0.01$ s. We observe that as K increases, $E_r(R_0)$ initially dwells at zero, then starts to increase, and gradually exceeds

¹Note that the actual total power needed should be scaled in accordance with the normalization $\rho(1 \text{ km}, f) = 1$.

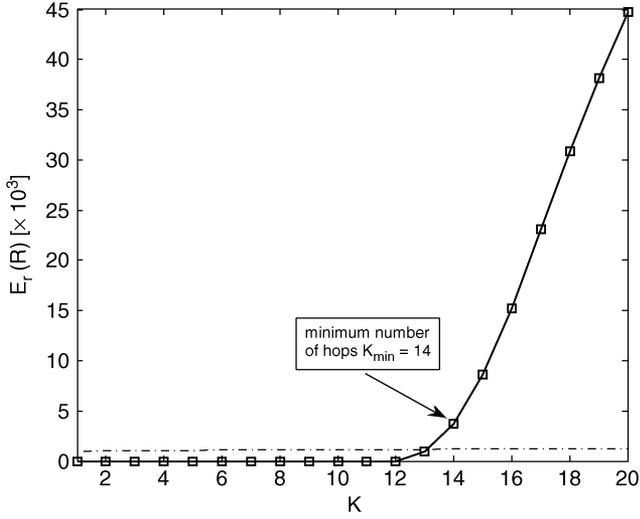


Fig. 8. Finding the minimal number of hops necessary to support prespecified rate $R_0 = 100$ kb/s and packet error probability $E_0 = 10^{-4}$. The curve with squares indicates the relationship between the number of hops K and the resulting random-coding error exponent $E_r(R_0)$, and the dashed-dotted curve indicates $(1/T_{\max}) \log(K/E_0)$ versus K . The first square above the dashed-dotted curve corresponds to the minimal number of hops, which is $K_{\min} = 14$ in this plot. System parameters are $d_{\text{tot}} = 10$ km, $P_{\text{tot}} = 40$ dB re μPa (normalized), and $T_{\max} = 0.01$ s.

$(1/T_{\max}) \log(K/E_0)$. The minimum number of hops turns out to be $K_{\min} = 14$ (i.e., hop distance $d_{\text{tot}}/K_{\min} \approx 714$ m), rather than 12 as in the idealized analysis of Fig. 7, which overlooked the effect of finite packet length on packet error probability. For network infrastructure and routing design, Fig. 8 suggests that the number of hops should be carefully chosen: using too few hops would fail to yield the required reliability for transmission; using too many hops tends to be wasteful since either the achievable reliability or the achievable rate rapidly exceeds what the network needs to provide.

Finally, we plot in Fig. 9 the minimum number of hops as we change the prespecified information rate from zero to around 200 kb/s, for the same system parameters as those of Fig. 8. The K_{\min} versus R relationship is piecewise integer valued from 1 to around 20, as the information rate increases. Such a plot yields a convenient tool for finding the number of hops required for a given rate. For example, if the prespecified information rate is $R_0 = 100$ kb/s, then Fig. 9 immediately indicates that the minimum number of hops is $K_{\min} = 14$.

VII. CONCLUSION

This paper presents an analysis of multihopping strategies for achieving high-rate transmission in underwater acoustic networking applications. As exemplified in earlier investigations (e.g., [11]), performance of signal detection can be dramatically improved through multihopping in underwater acoustic environments. In this paper, our analysis of information-theoretically achievable rates confirms such a benefit, and yields additional insights into the design of efficient coding schemes. As shown through a numerical study, transmission protocols with a spatial reuse factor as small as two typically strike the optimal balance between interference and rate scaling. Furthermore, when inter-hop interference cannot be safely ignored, performing water

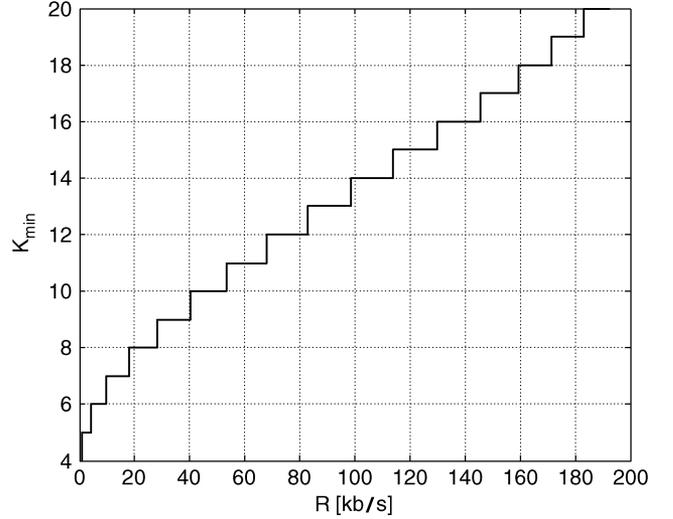


Fig. 9. Minimal number of hops as a function of the prespecified information rate R (in kilobits per second). System parameters are $d_{\text{tot}} = 10$ km, $P_{\text{tot}} = 40$ dB re μPa , $E_0 = 10^{-4}$, and $T_{\max} = 0.01$ s.

filling hop by hop is not optimal and the optimal signaling PSD should be computed taking interference into account. We develop a tool for determining the minimum number of hops for supporting prespecified rate and reliability, and exemplify its application through the random-coding exponential error bound. For practical systems, we note that it may be more relevant to evaluate the channel reliability function in an *ad hoc* fashion for the specific coding scheme used. Our numerical result indicates that in determining the minimum number of hops, the ideal assumption of infinitely long packets may yield overly optimistic estimate of the minimum number of hops, and it is usually necessary to take into account the coding block length versus decoding error probability tradeoff revealed by the channel reliability function.

APPENDIX

A. Proof of Proposition 1

Proof: We prove Proposition 1 by contradiction. Assume that there is a transmission protocol such that every node is more than $Q + 1$ hops away from its corresponding nearest interfering node. Denote the number of time slots for a packet to travel from a transmit node to a receive node by D . Consider the node \mathcal{N}_{Q+1} , and its $Q + 1$ upstream nodes from \mathcal{N}_Q back to \mathcal{N}_0 . Denote the time slots in which \mathcal{N}_{Q+1} is in the \mathbf{R} mode by $\{t_i^{\mathcal{N}_{Q+1}, \mathbf{R}}\}_{i=\dots, 0, 1, \dots}$. These time slots, on average, should occupy a $1/Q$ fraction of the total number of time slots. Consequently, the upstream node \mathcal{N}_Q , in time slots $\{t_i^{\mathcal{N}_{Q+1}, \mathbf{R}} - D\}_{i=\dots, 0, 1, \dots}$ has to be in the \mathbf{T} mode. At the same time, since by our assumption there is no interference from any node within $Q + 1$ hops away from \mathcal{N}_{Q+1} , nodes \mathcal{N}_{Q+1-l} for any $l = 2, \dots, Q + 1$, in time slots $\{t_i^{\mathcal{N}_{Q+1}, \mathbf{R}} - lD\}_{i=\dots, 0, 1, \dots}$, are not allowed to transmit, and, consequently, no reception is allowed.

Next, consider node \mathcal{N}_Q . Since it transmits in the time slots $\{t_i^{\mathcal{N}_{Q+1}, \mathbf{R}} - D\}_{i=\dots, 0, 1, \dots}$, it needs to use another set of time slots for reception, which we denote by $\{t_i^{\mathcal{N}_Q, \mathbf{R}}\}_{i=\dots, 0, 1, \dots}$. Consequently, these receiving time slots pose constraints on

the nodes \mathcal{N}_{Q+1-l} for $l = 2, \dots, Q + 1$. In particular, it follows that $\{t_i^{\mathcal{N}_{Q+1, \mathbf{R}}} - lD\}_{i=\dots, 0, 1, \dots}$ do not overlap with $\{t_i^{\mathcal{N}_{Q, \mathbf{R}}} - lD\}_{i=\dots, 0, 1, \dots}$.

As we continue the above reasoning over Q nodes from \mathcal{N}_{Q+1} to \mathcal{N}_2 , each node specifies a subset of time slots for its own reception without any interfering transmissions. These subsets do not overlap and each of them occupies a $1/Q$ fraction of the total number of time slots, so that their union is exactly the entire time line. Hence, we see that for node \mathcal{N}_0 , all of its time slots are prohibited from transmission, and, consequently, it is impossible to establish communication between \mathcal{N}_0 and \mathcal{N}_1 . This leads to a contradiction, and therefore, the statement of Proposition 1 is validated.

B. Proof of the Optimal Signaling PSD (9)

We will solve the following optimization problem:

$$\max_{S(f) \geq 0} \int_{f \geq 0} \log \left(1 + \frac{\rho(d, f) S(f)}{1 + S(f) \rho_I(Q, d, f)} \right) df \quad (16)$$

$$\text{s.t.} \int_{f \geq 0} S(f) df = P. \quad (17)$$

First, we observe that the objective function is concave in $S(f)$, and that the feasible set of $S(f)$ is convex. Therefore, we can use the method of Lagrange multipliers and the Karush–Kuhn–Tucker (KKT) condition.

Considering the Lagrangian

$$J = \int_{f \geq 0} \log \left(1 + \frac{\rho(d, f) S(f)}{1 + S(f) \rho_I(Q, d, f)} \right) df - \lambda \int_{f \geq 0} S(f) df \quad (18)$$

and taking its variation in $S(f)$, we obtain

$$\frac{\partial J}{\partial S(f)} = \frac{[\rho(d, f) + \rho_I(Q, d, f)]}{1 + [\rho(d, f) + \rho_I(Q, d, f)] S(f)} - \frac{\rho_I(Q, d, f)}{1 + S(f) \rho_I(Q, d, f)} - \lambda. \quad (19)$$

Hence, the stationary points ($\partial J / \partial S(f) = 0$) should satisfy

$$\rho_I(Q, d, f) [\rho(d, f) + \rho_I(Q, d, f)] [S(f)]^2 + [\rho(d, f) + 2\rho_I(Q, d, f)] [S(f)] + [1 - (1/\lambda)\rho(d, f)] = 0 \quad (20)$$

which has two solutions. Only if $0 < \lambda \leq \rho(d, f)$, (20) has one nonnegative solution (21) shown at the bottom of the page. If $\lambda > \rho(d, f)$, then the optimal $S(f)$ is given by the boundary $S(f) = 0$, according to the KKT condition. We thus establish the optimality of the solution.

$$S(f) = \frac{1}{2\rho_I(Q, d, f) [\rho(d, f) + \rho_I(Q, d, f)] \cdot \left\{ -[\rho(d, f) + 2\rho_I(Q, d, f)] + \sqrt{\rho^2(d, f) + (4/\lambda)\rho(d, f)\rho_I(Q, d, f) [\rho(d, f) + \rho_I(Q, d, f)]} \right\}} \quad (21)$$

C. Random-Coding Exponential Error Bound

Consider the capacity $C(d)$ given by (12), for a hop distance d and average power constraint P . Denote the width of the frequency range over which the receiver SINR is nonzero by W_e and call it the effective bandwidth. We can then define the effective SNR ρ_e as that which satisfies

$$W_e \log(1 + \rho_e) = C(d). \quad (22)$$

The rationale behind this definition is that we use a fictitious white Gaussian noise channel to replace the true channel, and since white Gaussian noise is the worst case noise, the achievable performance yielded by the fictitious channel serves as a lower bound.

Information theory asserts that for any packet length $T > 0$ and any rate $R < C(d)$, there exist a coding scheme and a corresponding maximum-likelihood decoding algorithm, such that the packet error probability is upper bounded by $E(R, T) = \exp[-T \cdot E_r(R)]$, in which the so-called random-coding error exponent $E_r(R)$ is given by the following [21].

- 1) If $R > C(d)$, $E_r(R) = 0$.
- 2) If R satisfies

$$W_e \log \left\{ \frac{1}{2} \left[1 + \frac{\rho_e}{2} + \sqrt{1 + \frac{\rho_e^2}{4}} \right] \right\} \leq R \leq C(d) \quad (23)$$

we have the parametric form

$$R = W_e \log \left(\beta + \frac{\rho_e}{1 + \alpha} \right) \quad (24)$$

$$E_r(R) = W_e [\log \beta + (1 + \alpha)(1 - \beta)] \quad (25)$$

where for each $0 \leq \alpha \leq 1$, β is given by

$$\beta = \frac{1}{2} \left\{ 1 - \frac{\rho_e}{1 + \alpha} + \sqrt{\left(1 - \frac{\rho_e}{1 + \alpha} \right)^2 + \frac{4\rho_e}{(1 + \alpha)^2}} \right\}. \quad (26)$$

- 3) If R satisfies

$$0 \leq R < W_e \log \left\{ \frac{1}{2} \left[1 + \frac{\rho_e}{2} + \sqrt{1 + \frac{\rho_e^2}{4}} \right] \right\} \quad (27)$$

we have

$$E_r(R) = W_e \log \left\{ \frac{1}{2} \left[1 + \sqrt{1 + \frac{\rho_e^2}{4}} \right] \right\} + W_e \left[1 + \frac{\rho_e}{2} - \sqrt{1 + \frac{\rho_e^2}{4}} \right] - R. \quad (28)$$

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