Packet Coding with Joint Power and Rate Control for Underwater Broadcast Networks

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ABSTRACT

We consider random linear packet coding for an underwater acoustic broadcast network. Joint power and rate control is considered as a means to overcome the effects of channel fading. Two main constraints are imposed in our optimization: (i) the transmit power cannot exceed a maximal value, and (ii) the number of coded packets should not exceed a maximum value. Under these constraints, we define two adaptation policies to adjust the transmit power and number of coded packets such that the average energy per bit is minimized. In the first case, the transmitter adapts in accordance with the average of the channel gains (average link rule) obtained as a feedback from each receiving node. In the second case, the transmitter adjusts its parameters in accordance with the lowest channel gain (worst link rule) among the receiving nodes. We show simulation results to quantify the energy savings available by employing the adaptation policies.

1. INTRODUCTION

Underwater acoustic networks are employed in a number of fields such as tsunami warning systems, ocean floor monitoring, off-shore oil platform monitoring, etc. Most underwater networks employ acoustics for communication and hence suffer from large latency due to the slow speed of sound in water (1500m/s). This large latency combined with the half duplex nature of most commercially available acoustic modems calls for system designs beyond the traditional ARQ techniques for underwater networks. In this paper, we address this issue with random linear packet coding for information broadcast in an underwater acoustic network.

A system employing packet coding buffers a block of M information-bearing packets at the transmitter and encodes them into a larger set of N coded packets to be transmitted. At each receiver, the original information bearing packets can be recovered from a subset of at least M of the N received packets. Since packet coding is applied at the packet level (as opposed to bit-level coding), this technique can be easily applied to most commercially available underwater acoustic modems. Random linear packet coding for a point-to-point link was explored in [1–5]. In [1], rateless coding was employed for reliable file transfer in underwater acoustic networks. It was shown that network performance is improved because the system used fewer feedback when compared to traditional ARQ techniques. Optimal schedules for random linear packet coding in half duplex links were explored in [2], which show that optimal number of coded packets can be obtained to minimize the average time (or energy) needed to complete the transmission of a block of packets. Random linear packet coding in the absence of a feedback link was introduced in [3], where the number of coded packets to transmit is determined such that the receiver can decode the original packets with a pre-specified reliability. In [4], the problem of joint optimization of mean throughput and packet drop rate in network coded systems was considered. It was found that a feedback-free packet coding approach provides better performance in terms of the mean throughput and packet drop rate.

In [5], a framework to combine adaptive power and rate control with random linear packet coding was described. Using average energy per bit as the figure of merit, it was shown an optimal number of coded packets exists that minimizes the average energy per bit, when employing adaptive power control. Similarly there exists an optimal transmit power that minimizes the average energy per bit when employing adaptive rate control.

Random linear packet coding for a broadcast network is considered in [6–8]. Optimal broadcasting policies for underwater acoustic networks based on random linear packet coding was investigated in [6], showing performance improvements over traditional ARQ techniques. In [7], random linear packet coding for broadcasting in time division duplexing channel was studied. It was shown that the packet coding approach outperforms optimal scheduling policies for broadcast in terms of the mean completion time. In [8], we provided a frame work to perform adaptive power control for a broadcast network.

In our earlier work [9], we provided adpatation policies for joint power and rate control with constrained resources for a point-to-point link. In this work, we aim to extend joint power and rate control to a broadcast network. We consider a network with a leader node that wants to broadcast information to multiple receiver nodes. Assuming a block fading channel model, the channel gain is decomposed into two parts: the large-scale slowly-varying part which admits feedback, and the small-scale fast-varying part which does not admit feedback, but influences the bit error rate performance. A feedback link is used to convey the large-scale fading information from the receiver to the transmitter. We define two constraints on the available resources: (a) the transmit power cannot exceed a maximum level and (b) the number of coded packets cannot exceed a maximum value. Under these constraints, we provide two adaptation policies: (a) the worst link rule, and (b) the average link rule, to perform joint power and rate control while minimizing the average energy per bit. Using the worst link rule, the transmit power and rate are adjusted in accordance with the link have the least channel gain, while the average of the channel gains are used in the average gain rule.

We further examine two separate cases: (i) the maximum number of coded packets is less than the optimal number of coded packets, and (ii) the maximum number of coded packets is more than the optimal number of coded packets. The optimal number of coded packets is that which minimizes the average energy per bit consumption in the absence of any constraints. We provide simulation results to quantify the energy savings available in each of the cases.

The rest of the paper is organized as follows. We present the system model and describe the system parameters in Sec. 2. We present the optimization criterion and the adaptation policy for the two cases in Sec. 3. We present the simulation results in Sec. 4 and summarize the conclusions in Sec. 5.

2. SYSTEM MODEL

We consider an underwater network with a leader node that wishes to broadcast information to D receiver nodes. In each broadcast cycle, the leader buffers a block of M packets and encodes them into $N \geq M$ packets for transmission. Each coded packet contains N_b bits, K_b of which are the information bits and $N_b - K_b$ are the overhead bits. The channel bit rate is R_b, and the effective information rate is K_b/T_p. The duration of each packet is thus $T_p = N_b/R_b$. After every block of packets the leader waits for a feedback from each of the receivers which contains the channel gain G_i between the leader and the i^{th} receiver node.

The leader transmits at a power P_T over a channel with large-scale channel gain G_i. The signal power at the i^{th} receiver is $P_{R,i} = G_i P_T$. The signal-to-noise ratio at the i^{th} receiver is given by $\gamma_i = P_{R,i}/P_N = G_i P_T/P_N$, where P_N is the noise power. For this analysis, we assume a block fading channel, where the channel gain remains constant over a block of packets and changes from one block to another. The coherence time of the channel T_C is the duration for which the channel gain remains constant and is assumed to be same for each link. Since the large-scale channel gain is varying slowly over time, it can be fed back to the transmitter for adapting the transmit power and rate. In this analysis, we model the large scale channel gain as log-normally distributed [10], i.e., $10\log_{10} G \sim \mathcal{N}(\bar{g}, \sigma_g^2)$. For the purposes of this work, we assume that the receiver nodes are equidistant from the leader and hence all the links have the same $\bar{g}$ and σ_g^2.

The probability of bit error (BER) is a function of the signal-to-noise ratio and is represented by $P_e(\gamma)$. The corresponding packet error rate for each link is given as $P_E(\gamma_i) = 1 - (1 - P_e(\gamma_i))^{K_b}$.

We define the probability of successful decoding P_s as the probability that at least M out of the N transmitted packets are received without errors. It is given by

$$P_s(\gamma) = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma))^m P_E^{N-m}(\gamma) \qquad (1)$$

We wish to maintain a pre-defined success rate at the receiver and this value is denoted by P_s^*. We can now define the outage probability as the probability that $P_s(\gamma)$ falls below a pre-defined value P_s^*, i.e., $P_{out} = \{P_s(\gamma_i) < P_s^*\}$. Under the assumption of log-normal distribution, the probability of outage is given by

$$P_{out} = Q(\frac{\bar{g} - 10\log_{10} G_{out}}{\sigma_g}) \qquad (2)$$

and the outage gain is thus given by

$$G_{out} = 10^{\frac{\bar{g} - \sigma Q^{-1}(P_{out})}{10}} \qquad (3)$$

Energy conservation is an important requirement in underwater deployments because power is limited and saving energy directly impacts the duration of each deployment. The average energy per bit is given by

$$\bar{E}_b = \frac{1}{R_b} \frac{E\{NP_T\}}{P_s^* M} \qquad (4)$$

On the one hand, increasing the transmit power directly increases the average energy per bit, but on the other hand it leads to higher signal-to-noise ratio, and hence fewer coded packets would be necessary. It is this trade-off we wish to exploit and find the optimal values for the transmit power and the number of coded packets.

In the interest of practical systems where resources are limited, we impose two main constraints on our optimization.

1. The transmit power is limited to $P_{T,max}$ and is dictated by hardware system constraints or by the total budget.

2. The number of coded packets is limited to maximal value N_{max}. N_{max} is determined so as to satisfy all of the following requirements: (i) a batch must not last longer than a value dictated by the coherence of the channel gain T_C; (ii) decoding delay must not exceed a maximum tolerable value T_d, and (iii) average bit rate must not fall below a tolerable minimum $R_{b,min}$. N_{max} is thus given as

$$N_{max} = \begin{cases} \frac{T_C}{T_p}, & T_C \leq T_d, \frac{R_b P_s^* M}{T_C/T_p} \geq R_{b,min} \\ \frac{T_d}{T_p} & T_C > T_d, \frac{R_b P_s^* M}{T_d/T_p} \geq R_{b,min} \\ \frac{R_b P_s^* M}{R_{b.min}}, & \text{otherwise} \end{cases} \qquad (5)$$

3. OPTIMIZATION PROCEDURE

The relationship between P_s and P_E is shown in Fig. 1 for different values of N. As described earlier, we wish to have a pre-defined reliability P_s^* at the receiver. By setting a desired P_s^* (horizontal line), we can see that for every candidate N, there is a corresponding value $P_E^* = P_E^*(N)$. This $P_E^*(N)$ points to a particular value of the SNR, $\gamma^*(N)$ for a specific small scale fading type and a modulation/coding/diversity scheme. For purposes of this work, we assume differentially coherent detection is employed with no

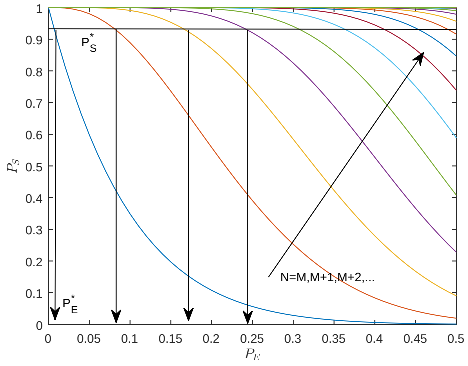


Figure 1: Probability of successful decoding vs. probability of packet error for different values of N.

coding or diversity, and a small scale fading that is Rician distributed. Our analysis does not change with changes in any of these assumptions, however, numerical results do.

For a given channel gain G, the transmit power needed to achieve $\gamma^*(N)$ is

$$P_T^* = \gamma^*(N) P_N / G \tag{6}$$

Fig. 2 shows the product NP_T^* as a function of N for different values of the gain G. As shown in [9], tt can seen that the value of N which minimizes NP_T^* is the same as that which minimizes $N\gamma^*(N)$ and hence does not depend on the channel gain G. We denote this value by N_{opt} as it is the value that minimizes the average energy per bit $\bar{E}_b$ in the absence of any constraints.

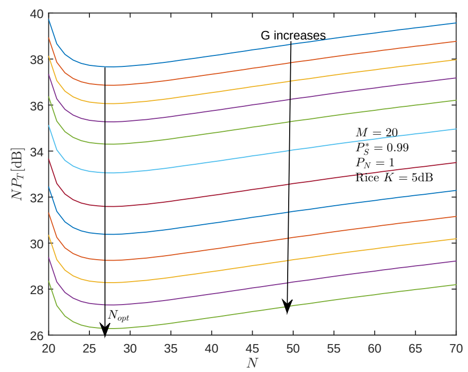


Figure 2: Plot of NP_T^* vs N. To minimize the average energy per bit, the number of coded packets and the transmit power should be chosen such that the product NP_T^* is minimized (subject to system constraints $P_{T,max}, N_{max}$).

Depending on the relationship between N_{max} and N_{opt} , two cases emerge for the optimization procedure. The first case is one in which N_{max} is less than N_{opt}, i.e., $N_{max} \leq N_{opt}$, and the second case occurs when $N_{max} > N_{opt}$. We analyze each of these cases in detail in the following sections and provide the adaptation policies.

3.1 Case 1: $N_{max} \leq N_{opt}$

In this case, the number of coded packets has to be fixed at $N = N_{max}$ since N_{max} is less than N_{opt}. The transmit power can be adapted according to two rules: (a) worst link rule, and (b) average link rule.

When employing the worst link rule, the leader adapts its transmit power P_T in accordance with the link that has the worst (least) channel gain. The worst channel gain can be computed as

$$G_{min} = \min_{G_i : G_i \geq G_{out}} G_i \tag{7}$$

The adaptation policy can be expressed as

$$P_T = \begin{cases} \gamma^*(N_{max}) P_N / G_{min}, & G_{min} > G_{out} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

When employing the average link rule, the leader adapts its transmit power P_T in accordance to the average of the channel gains on each link. The average channel gain is given as

$$G_{av} = \frac{1}{D} \sum_{G_i : G_i \geq G_{out}} G_i \tag{9}$$

The adaptation policy can now be expressed as

$$P_T = \begin{cases} \gamma^*(N_{max}) P_N / G_{av}, & G_{av} > G_{out} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

For the purposes of performance evaluation, we have to compare the adaptation scheme with a benchmark system that operates that operates at fixed power and rate. In the absence of adaptive control, transmit power is kept fixed at some $P_{T,fix}$, and the number of packets is fixed at N_{fix}. The resulting SNR, $\gamma = P_{T,fix} G / P_N$, changes with the gain, and so do the probabilities P_E and P_s. Outage occurs when $\gamma < \gamma^*(N)$, and the probability of outage is

$$\begin{aligned} P_{out} &= P\{\gamma < \gamma^*(N)\} = P\{G < \underbrace{\gamma^*(N) P_N / P_{T,fix}}_{G_{fix}}\} \\ &= Q\left(\frac{\bar{g} - 10\log_{10} G_{fix}}{\sigma_g}\right) \end{aligned} \tag{11}$$

The power needed to keep the outage at a pre-specified level P_{out} is thus $P_{T,fix} = \gamma^*(N) P_N / G_{fix}$, where $10\log_{10} G_{fix} = \bar{g} - \sigma Q^{-1}(P_{out})$.

3.2 Case 2: $N_{max} > N_{opt}$

An additional degree of freedom is available for the case $N_{max} > N_{opt}$, making the adaptation policy somewhat more involved. For this case, there are three regions of operation, one in which the system is shut off and two in which the transmission is active. When the system is active, there are further two regions of operation, a high gain region in which the number of packets is kept fixed at $N = N_{opt}$ and power control is performed, and a low gain region where the power is kept fixed at $P_T = P_{T,max}$ and rate control is performed.

As stated earlier, when the channel gain is high, the number of coded packets is fixed at $N = N_{opt}$, and the power is varied. As the gain diminishes, the break point occurs at

$$G_{break} = \gamma^*(N_{opt}) P_N / P_{T,max} \tag{12}$$

If the gain drops below this value the power is kept at $P_T = P_{T,max}$ and rate adaptation is performed. The outage point occurs at

$$G_{out} = \gamma^*(N_{max}) P_N / P_{T,max} \tag{13}$$

If the gain drops below this value, there is no solution for (N, P_T) that satisfies the required P_s^*, and transmission is shut off.

The adaptation policy for the case $N_{max} > N_{opt}$ using the worst link rule is

$$N = \begin{cases} \gamma^*_{inv}(\frac{P_{T,max} G_{min}}{P_N}), & G_{break} > G_{min} > G_{out} \\ N_{opt}, & G_{min} \geq G_{break} \\ 0 & \text{otherwise} \end{cases}$$

$$P_T = \begin{cases} P_{T,max}, & G_{break} > G_{min} > G_{out} \\ \frac{\gamma^*(N_{opt}) P_N}{G_{min}} & G_{min} \geq G_{break} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The adaptation policy for the case $N_{max} > N_{opt}$ using the average link rule is

$$N = \begin{cases} \gamma^*_{inv}(\frac{P_{T,max} G_{av}}{P_N}), & G_{break} > G_{av} > G_{out} \\ N_{opt}, & G_{av} \geq G_{break} \\ 0 & \text{otherwise} \end{cases}$$

$$P_T = \begin{cases} P_{T,max}, & G_{break} > G_{av} > G_{out} \\ \frac{\gamma^*(N_{opt}) P_N}{G_{av}} & G_{av} \geq G_{break} \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

4. SIMULATION RESULTS

In this section we evaluate the performance of each technique using simulation. We consider a network of $D = 5$ nodes that are equidistant from each other. We further assume that the all links have a channel gain that is log-normal distributed with mean gain $\bar{g} = -10$dB.

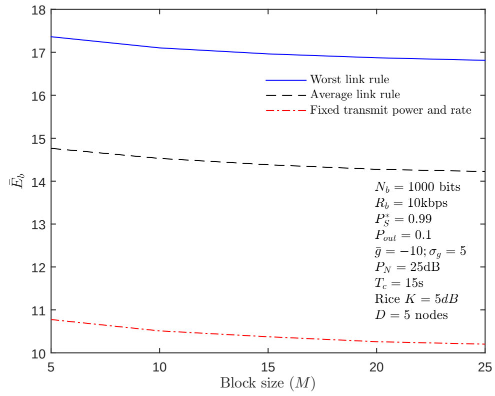
The performance of the adaptive power and rate control strategies for *Case 1* is summarized in Fig. 3. We can see an average savings of about $6 - 12$ dB by employing the average link rule and $2 - 6$dB using the worst link rule. The average link rule does lead to a higher outage ($\sim 26\%$) than the system is designed for (10%) because the leader is being conservative and adapting to the average of the links, thus starving some of the nodes. While employing the worst link rule, the measured outage was $\sim 12\%$ because the leader also transmits to those nodes which have a very poor link ($G_i < G_{out}$) and hence were not considered during the calculation of G_{min}.

Fig. 3a shows the average energy per bit as a function of the block size M. Although larger block sizes leads to lower average energy per bit, one has to keep in mind that the block sizes cannot become so large that they violate the block fading model. The value of the block size M is thus a design choice made based on the coherence time of the channel. Fig. 3b shows the average energy per bit as a function of the packet size N_b. Despite lowering the average energy per bit, smaller packet sizes will not be favored in the interest of higher throughput. Fig. 3c shows the average energy per bit as a function of the fading parameter σ_g. The savings available by employing adaptive power and rate control increases with an increase in the channel fading.

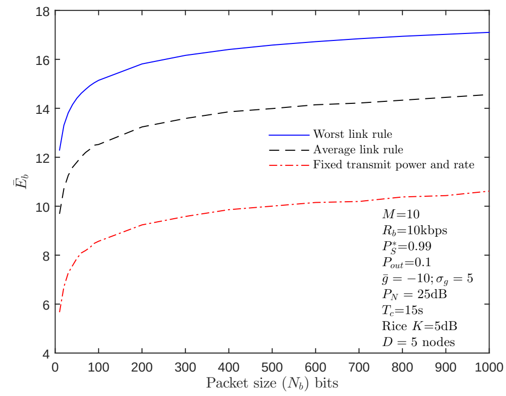
The performance of the system for $N_{max} > N_{opt}$ is shown in Fig. 4. As in the previous case, we plot the energy per bit as a function of block size (Fig. 4a), packet size (Fig. 4b), and channel fading parameter σ_g (Fig. 4c). Higher energy savings are available in *Case 2* because the leader now has an additional degree of freedom for its optimization.

5. CONCLUSION

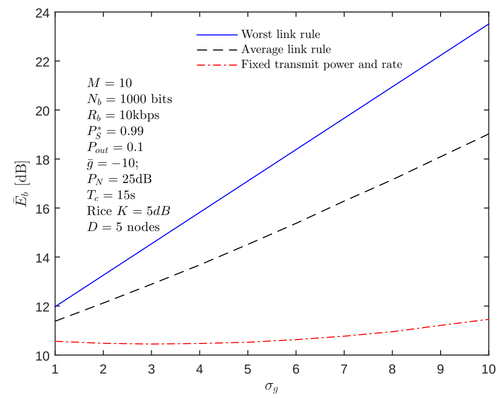
We analyzed joint power and rate control for an underwater broadcast network employing random linear packet



(a) Average energy per bit as a function of the block size M. Although larger blocks are favorable as they lead to lower average energy per bit, it is important to keep in mind that the block fading model should not be violated.

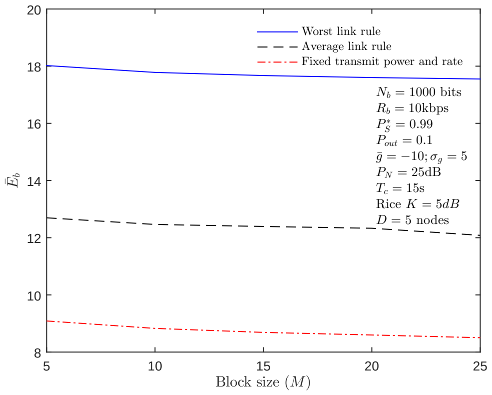


(b) Average energy per bit as a function of the number of bits in each packet. Although smaller packets use less energy per bit, the average throughput will suffer with smaller packet sizes.

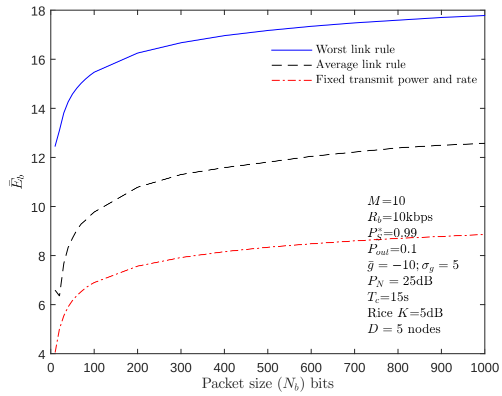


(c) Average energy per bit as a function of the standard deviation σ_g.

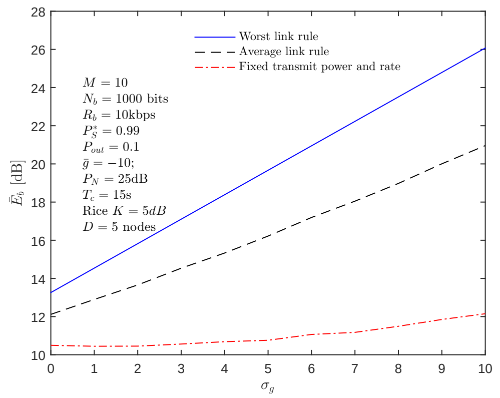
Figure 3: Performance plots for *Case 1* where $N_{max} \leq N_{opt}$. $R_{b,min}$ was chosen to be 7kbps for this case.



(a) Average energy per bit as a function of the block size M. As in *Case 1*, larger block sizes are favorable as long as they do not violate the block fading assumption.



(b) Average energy per bit as a function of the packet size N_b. Although smaller packets lead to lesser average energy per bit, design choice for should also consider the average throughput.



(c) Average energy per bit as a function of the channel fading parameter σ_g. As seen in *Case 1*, higher savings are available when the channel fading increases.

Figure 4: Performance plots for *Case 2* where $N_{max} > N_{opt}$. $R_{b,min}$ was chosen to be 3kbps for this case.

coding. We propose a system design based on reliability to improve the overall network efficiency which is otherwsie hampered by the long propagation delays of underwater acoustics.

We propose two adaptation policies: (a) worst link rule where the transmitter adapts in accordance with the link that has the least channel gain, and (b) average link rule where the transmitter adapts in accordance to the average of the channel gains on each link. We show using simulations that an average energy savings of $6 - 12$dB, and $2 - 6$dB is available by employing the worst link rule and average link rule, respectively.

Our future work will focus on providing analytical solutions for the average energy per bit when employing the two different rules. We will further extend our analysis to a network where the mean of the channel gain is not same on each link and analyze its impact on the energy savings.

Acknowledgments

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