On Capacity of a Class of Acoustic Channels

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***Abstract*— We use a per-path Rician fading model to analyze the capacity of acoustic channels, and provide experimental data to confirm the practicality of simulation results. Three power allocation policies are considered: water-filling, uniform power allocation across selected, channel-favored frequencies ("on-off" carriers in an OFDM system), and uniform power allocation across all carriers irrespective of the channel ("all-on"). The impact of channel estimation errors and feedback delay is taken into account. The three policies are found to differ little in terms of the achievable rate, and to suffer similar loss from imperfect channel knowledge at the receiver (about 1 bps/Hz loss from 4 bps/Hz at 20 dB for the experimental channel). Unlike water-filling and on-off power allocation, which are additionally sensitive to outdated channel feedback, all-on uniform power allocation does not require feedback and is simple to implement, thus emerging as a justified practical solution.**

***Index Terms*—Underwater acoustic communications, channel capacity, information rate, lower bound, Rician fading, water-filling, power allocation, OFDM.**

I. Introduction

In contrast to radio communications, where capacity issues are well understood for point-to-point links (see e.g. [1], [2]), the fundamental question of acoustic channel capacity has been an elusive one, mainly because of the lack of well-established statistical channel models. Early work by Kwon and Birdsall [3], and Leinhos [4], which addressed the theoretical aspects, was followed by Kilfoyle et al. who performed the first experimental analysis [5], [6]. More recently, Radosevic et al. [7], [8], and Socheleau et al. [9] have focused on a Rician channel model supported by experimental measurements.

The objective of our present work is threefold. First, we wish to re-visit the fundamental question of acoustic channel capacity and the rate achievable when the channel is not known but only estimated/predicted at the receiver/transmitter. We do so within the framework of orthogonal frequency division multiplexing (OFDM), which lends itself to practical implementation of related power- and bit-loading techniques [10]. We perform two types of analysis, one based on the experimental data collected during the 2010 mobile acoustic communications experiment (MACE'10), and another based on a recently developed statistical channel model [11], where each propagation path is described as a non-zero-mean, first-order auto-regressive complex-Gaussian random process. Second, we investigate different power allocation strategies, including (i) water-filling; (ii) on-off power allocation [12], [13], where power is allocated in equal amounts but only to those carriers whose SNR is above the water-filling threshold, and (iii) an unknown-channel strategy where power is simply allocated to all the carriers in equal amounts. Finally, we address the issue of imperfect channel knowledge in light of (a) pilot overhead needed to estimate the channel at the receiver and (b) fundamental propagation delay that affects feedback, limiting the transmitter to operate only with outdated channel estimates. Our results reflect the actual throughput after taking the guard intervals and pilot overhead into account.

The paper is organized as follows. In Sec.II we formally define the power allocation strategies, their respective capacities, and the corresponding lower bounds on the mutual information rate achievable in the presence of channel estimation errors (to which we shortly refer as "rate"). In Sec.III we discuss channel estimation and feedback strategies, and the impact of delay. Sec.IV is devoted to numerical results that quantify the achievable rate for the considered class of acoustic channels. Conclusions are summarized in Sec.V.

II. Achievable Rate

We consider a channel described (at some time) by a transfer function $H(f)$, whose values at carrier frequencies $f_k = f_0 + k\Delta f, k = 0, \dots K-1$ are denoted by $H_k = H(f_k)$ and assumed to be constant over a subband of width $\Delta f = 1/T$. The signal received on the k-th carrier frequency is modeled as

$$y_k = \sqrt{P_k} H_k d_k + z_k \tag{1}$$

where P_k is the power allocated to the k-th carrier, d_k is the unit-variance information signal transmitted on this carrier, and z_k is zero-mean, circularly symmetric Gaussian noise of variance $\sigma^2_{z_k}$. The total power allocated to the system is P_{tot}, and the total bandwidth is $B = K\Delta f$.

The capacity of this system is given by Shannon's formula[1]

$$C = \Delta f \sum_{k=0}^{K-1} \log_2 \left(1 + \frac{P_k |H_k|^2}{\sigma^2_{z_k}} \right) \text{ [bps]} \tag{2}$$

The power allocation policy which maximizes the capacity is specified by the water-filling rule,

$$P_k = \begin{cases} \nu - \frac{\sigma^2_{z_k}}{|H_k|^2}, \text{ when } |H_k|^2 > \frac{\sigma^2_{z_k}}{\nu} \\ 0, \text{ otherwise} \end{cases}, \; k = 0, \dots K-1 \tag{3}$$

where the parameter ν is determined such that

$$\sum_{k=0}^{K-1} P_k = P_{tot} \tag{4}$$

The power allocation used here requires both the transmitter and receiver to know the channel. In practice, estimates $\hat{H}_k$ can be formed at the receiver and passed back to the transmitter,

[1] Alternatively, the capacity can be expressed as C/B [bps/Hz].

where they are used to implement a power allocation policy. We will consider two such policies: one based on water-filling, and another based on an ad-hoc on-off rule. In both cases, channel estimation is accomplished using known pilot signals, for which K_p carriers are reserved in advance. The remaining $K-K_p$ carriers are used for data (information) and indexed by the set $\mathcal{K}_d$. Setting aside a fraction α of the total power for the pilots, $(1-\alpha)P_{tot}$ is left for distribution across the data carriers. The two power allocation policies are specified below.

1) Imperfect water-filling: Power is allocated to the data carriers according to the rule (3), but with channel estimates $\hat{H}_k$ replacing the unknown values H_k:

$$P_k = \begin{cases} \nu - \frac{\sigma^2_{z_k}}{|\hat{H}_k|^2}, \text{ when } |\hat{H}_k|^2 > \frac{\sigma^2_{z_k}}{\nu} \\ 0, \text{ otherwise} \end{cases}, \ k \in \mathcal{K}_d \tag{5}$$

and the water level ν is determined such that

$$\sum_{k\in\mathcal{K}_d} P_k = (1-\alpha)P_{tot} \tag{6}$$

Note that water-filling may render some carriers with no power. These are the carriers whose frequencies are not favored by the channel. We refer to them as "bad carriers," while the remaining carriers that are used to send data are called "good carriers."

2) On-off uniform power allocation: Using the value of ν found under the previous policy, the available power $(1-\alpha)P_{tot}$ is allocated in *equal* amounts P_d to the good carriers, while nothing is given to the bad carriers [12], [13]:

$$P_k = \begin{cases} P_d, \text{ when } |\hat{H}_k|^2 > \frac{\sigma^2_{z_k}}{\nu} \\ 0, \text{ otherwise} \end{cases}, \ k \in \mathcal{K}_d \tag{7}$$

The bit rate achievable when only the channel estimates are available can be gauged from the lower bound on mutual information [2],[2]

$$R = \frac{T}{T+T_g}\cdot\Delta f \sum_{k\in\mathcal{K}_d} \log_2\left(1+\frac{P_k|\hat{H}_k|^2}{\sigma^2_{z_k}+P_k\sigma^2_{\Delta H_k}}\right) \tag{8}$$

where $\sigma^2_{\Delta H_k} = E\{|\Delta H_k|^2\}$ is the variance of the channel estimation error $\Delta H_k = \hat{H}_k - H_k$, and the factor $T/(T+T_g)$ accounts for the multipath guard time T_g inserted between successive blocks of K carriers. The above expression is valid when minimum mean squared error channel estimation is employed at the receiver, such that the error ΔH_k is orthogonal to the estimate $\hat{H}_k$.

When the channel is randomly varying, so are the capacity C and the rate R. To account for different channel realizations, one can use the notion of average capacity $\bar{C}$, or outage capacity $C_{P_{out}}$, which is defined for a given outage probability P_{out} through $P_{out} = P\{C \le C_{P_{out}}\}$. Similar definitions apply to the rate.

[2]We will refer to this quantity shortly as "rate."

III. Channel Estimation and Delay

For time-varying channels, we distinguish between two effects of delay, one occurring at the receiver and another at the transmitter (outdated feedback). The first effect is present when the receiver does not compute a new channel estimate in every block, but uses one block's estimate to predict the channel for several blocks. This is done so as to reduce the total pilot overhead. Delayed feedback causes the transmitter's estimate (which is used to allocate the power) to be outdated even if the receiver conveys its instantaneous estimate. Propagation delay thus presents a fundamental limitation, and one expects it to play a dominant role in an acoustic channel due to the low propagation speed.

To specify the rate, let us denote by D_r the number of blocks over which the receiver makes channel predictions, and let D_t be the number of blocks involved in the delayed feedback. At the receiver, a new estimate $\hat{H}_k(n_0)$ is made at times $n_0 = 0, D_r, 2D_r$, etc., while the predictions $\breve{H}_k(n_0+m)$ are made in-between, for $m = 1, \ldots D_r - 1$. Hence, $\breve{H}_k(n_0) = \hat{H}_k(n_0)$ and $\breve{H}_k(n_0+1)$, etc., are derived from $\hat{H}_k(n_0)$. The resulting series $\breve{H}_k(n)$ is also known to the transmitter, but with a delay. The power $P_k(n)$ received during the n-th block has thus been allocated based on the prediction made on $\breve{H}_k(n-D_t)$.

Looking at a given frame of D_r blocks, pilots are assigned in the first block (n_0), while the remaining blocks use all the carriers for data. Channel mismatch is thus due to both the estimation error made in the first block and subsequent change in the channel. The instantaneous rate (c.f. 8) is

$$\underbrace{\tilde{R}(n)}_{n=n_0+m} = \frac{1}{T+T_g}\sum_{k\in\mathcal{K}_d(m)} log_2\left(1+\frac{P_k(n)|\breve{H}_k(n)|^2}{\sigma^2_{z_k}+P_k(n)\sigma^2_{\Delta H_k}(m)}\right) \tag{9}$$

where $\sigma^2_{\Delta H_k}(m) = E\{|H_k(n_0+m) - \breve{H}_k(n_0+m)|^2\}$, and the set $\mathcal{K}_d(m)$ equals the set of data carriers $\mathcal{K}_d$ when $m=0$, or all the carriers when $m \neq 0$. Averaged over one frame, the instantaneous rate is[3]

$$R(n_0) = \frac{1}{D_r}\sum_{m=0}^{D_r-1}\tilde{R}(n_0+m) \tag{10}$$

This rate is a random variable whose statistics indicate the corresponding outage rate $R_{P_{out}}$, or average rate $\bar{R} = E\{R(n_0)\}$ taken over multiple channel realizations.

A. Channel model

To assess the impact of delay on the achievable rate, time-evolution of the channel has to be modeled. We focus on a statistical channel model with P propagation paths, where each path is characterized during the n-th block by a random gain $h_p(n)$ and delay $\tau_p(n)$. The corresponding transfer function is

$$H_k(n) = \frac{1}{\sqrt{A_k}}\sum_p h_p(n)e^{-j2\pi f_k\tau_p(n)} \tag{11}$$

and the factor A_k is introduced to account for attenuation due to spreading and absorption (assumed same for all the paths).

[3]An additional loss factor should be included for half-duplex channels.

The path gains are modeled as independent, first-order auto-regressive processes[4]

$$[h_p(n) - \bar{h}_p] = \rho_p[h_p(n-1) - \bar{h}_p] + \chi_p(n) \quad (12)$$

where $\bar{h}_p = E\{h_p(n)\}$ is the mean value of the gain, the variance is $\sigma_p^2 = E\{|h_p(n) - \bar{h}_p|^2\}$, and $\chi_p(n) \sim \mathcal{CN}(0, (1-\rho_p^2)\sigma_p^2)$ is the process noise which is uncorrelated with $h_p(n-1)$ as well as across p. The one-step correlation coefficient ρ_p is related the Doppler spread B_p of the p-th path via $\rho_p = e^{-\pi B_p(T+T_g)}$. Time-evolution of the path delays is modeled as

$$\tau_p(n) = \tau_p(n-1) - a_p \cdot (T + T_g) \quad (13)$$

where a_p is the Doppler factor that captures motion-induced time scaling on the p-th path.[5]

B. Channel estimation

In practical OFDM systems, channel estimation is often performed in the equivalent discrete-time domain, using equi-spaced pilot carriers of equal power. The estimator targets sample-spaced channel *taps* which stand in a discrete Fourier relationship with the coefficients $H_k(n)$. Assuming that performance is dominated by J significant taps ($P \leq J \leq K_p$) whose locations are correctly determined by a sparse channel estimation method, and that $\sigma_{z_k}^2 = \sigma_z^2$, the resulting error variance is (see [14] for details)

$$\sigma_{\Delta H_k}^2(0) = J\sigma_z^2/\alpha P_{tot} \quad (14)$$

1) Illustration–special case: To gain insight into the effect of delay, one can look at an artificial example of a channel whose paths have fixed delays that coincide with integer multiples of T/K ($J = P$), and fade at the same rate $\rho_p = \rho$. Channel predictions are then made simply as $[\breve{H}_k(n_0+m) - \bar{H}_k] = \rho^m[\hat{H}_k(n_0) - \bar{H}_k], m = 1, \ldots D_r - 1$, and the power $P_k(n)$ is allocated according to $\bar{H}_k + \rho^{D_t}[\breve{H}_k(n - D_t) - \bar{H}_k]$. With equal noise power across carriers, this special case yields

$$\sigma_{\Delta H_k}^2(m) = \rho^{2m}\sigma_{\Delta H_k}^2(0) + (1-\rho^{2m})\sum_p \sigma_p^2 \quad (15)$$

The first term in the above expression reflects the noise-induced channel estimation error made in the pilot block, while the second term reflects the prediction error caused by the channel dynamics. The variance (15) can be substituted directly into the expressions (9) and (10) for the rate. The trade-off between the number of pilots and the channel estimation errors then becomes clear: as D_r increases, the total overhead decreases (K_p pilots cover $(K - K_p) + (D_r - 1)K$ data carriers), but every new channel prediction brings a stronger error. In the next section, we will quantify the relevant measures of rate using a channel model in which the paths fade at the same rate, but are not sample-spaced.

[4]Path independence follows from the fact that reflection points at which scattering occurs are sufficiently far apart. Path independence does not imply independence of equivalent sample-spaced taps (an assumption that is often made in the literature, e.g. [1]).

[5]Depending on the particular circumstances, Doppler scaling factors can be treated as deterministic or random, time-invariant or time-varying, known or unknown.

IV. RESULTS

Given the candidate power allocation policies, the question arises as to what performance can they deliver in terms of the rate, and how does this performance compare with the channel capacity. On the one hand, we have the imperfect water-filling which aspires to achieve the optimum (but ignores the penalty of channel estimation errors), while on the other hand we have on-off power allocation which does not follow any optimization incentive, but is simpler to implement and operate. Finally, we have the unknown-channel (and thus feedback-free) policy in which power is allocated equally to all carriers.

The results presented below are obtained using simulation and experimental data. The experimental data were recorded in a 100 m deep, 3-7 km long mobile channel, with 256 carriers occupying 10 kHz-15 kHz acoustic band (for details of deployment, see [15]). The results are based on 1664 OFDM blocks transmitted over a period of 3.5 hours. The guard interval is $T_g = 16$ ms (block duration is $T = 51$ ms). The simulated channel follows the model of Sec.III-A, where the average path gains and path delays are selected according to the channel geometry that matches the experimental one.[6] We select 50 different channel responses (which vary slightly in the placement of transmitter and receiver) and add random time-variation over the duration of 100 OFDM blocks (5000 blocks in total). To describe the variation, we use the notion of average Rician K-factor as introduced in [16], $\bar{K} = \sum_p \bar{h}_p^2 / \sum_p \sigma_p^2$. The noise profile is assumed to be constant over the bandwidth. The SNR is defined as SNR=$P_{tot}\sum_p(\bar{h}_p^2 + \sigma_p^2)/K\sigma_z^2$. The results are based on $K_p = K/4$ channel estimation pilots and pilot power ratio α=1/4, except for the genie-aided benchmark cases labeled "ideal channel knowledge," where no pilots are needed (data on all carriers).

Fig.1 shows the cumulative distribution function of the rate (8) calculated using simulated as well as experimentally measured channels. We note that the experimental channel matches closely with $\bar{K} = 2$. Different curves on this plot can be used to determine the outage rate for different $\bar{K}$.

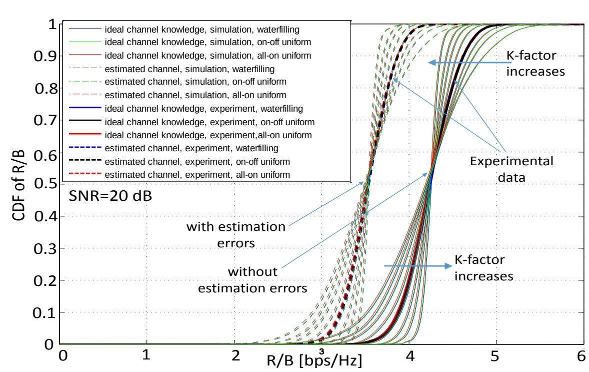

Fig. 1. Cumulative distribution function of the rate (8). The average K-factor takes values 0, 0.5, 1, 2, 4, 8 , 16, 64.

[6]The average multipath profile is characterized by the mean gain magnitudes 1, 0.9, 0.5, 0.45, 0.4, 0.3, 0.1; delays 0, 0.4, 1.7, 3, 5.5, 7.6, 11.5 ms, and K-factors 1, 0.2, 0.5, 0.15, 0.15, 0.07, 0.1, all relative to the first arrival.

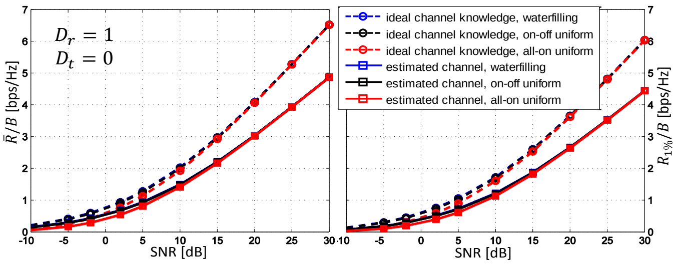

Fig. 2. Average rate (left) and 1% outage rate (right), calculated using experimental data. This result corresponds to instantaneous feedback.

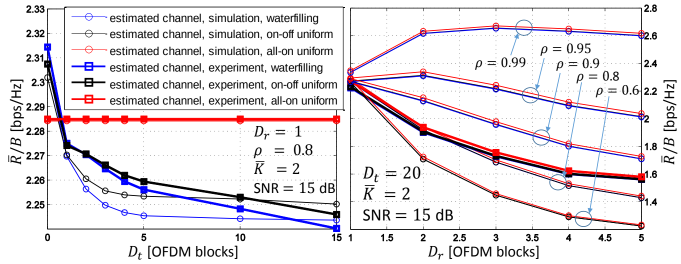

Fig. 3. Average rate as a function of feedback delay (left) and receiver's prediction window (right).

Next, we compare the power allocation strategies in terms of the average rate $\bar{R}$ and and the outage rate $R_{P_{out}}$. Fig.2 shows $\bar{R}$ (left) and $R_{1\%}$ (right) vs. SNR, calculated from experimental data. Similar results are obtained in simulation. Different power alloction strategies are seen to perform with negligible difference on this channel, providing about 4 bps/Hz at the SNR of 20 dB under ideal channel knowledge. Channel estimation errors impact all the strategies in a very similar manner, causing a loss of about 1 bps/Hz at 20 dB.

This result takes into account only the imperfect channel knowledge at the receiver, i.e. it assumes an instantaneous feedback. In the presence of feedback delay, the performance of all-on uniform allocation will be the same, while the other two strategies can only perform worse. This fact is illustrated in Fig. 3 (left). In the absence of feedback delay, waterfilling and on-off policies offer a small advantage; however, the situation is equalized and eventually reversed with outdated feedback, as the "good" and "bad" carriers are no longer identified correctly. These facts speak strongly in favor of using the simple, feedback-free, all-on uniform power allocation.

The effect of receiver's prediction window D_r is illustrated in Fig. 3 (right). As D_r increases, the pilot overhead is reduced, but the penalty of prediction error becomes significant (c.f. 15), eventually reducing the average rate. Thus, there exists an optimal choice of D_r for a given channel (e.g. D_r=3 when ρ=0.99).

Channel estimation plays a key role in the achievable rate (with uniform, or any other power allocation policy). In practice, the trade-off between accurate channel estimation and pilot overhead can additionally be negotiated by employing decision-directed tracking algorithms, whose implementation is much simplified under uniform power allocation.

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V. Conclusion

A capacity analysis was presented for acoustic channels in which each path is modeled as an auto-regressive, non-zero-mean complex-Gaussian process. Numerical results, obtained via simulation and confirmed using experimentally measured channels, indicate the average achievable rate (lower bound) on the order of 3 bps/Hz at the SNR of 20 dB (1% outage rate is lower by 0.5 bps/Hz) in the presence of channel estimation errors. On comparing three power allocation policies – waterfilling, on-off uniform and all-on uniform – the first two were found to offer little or no advantage on this channel. The simple, unknown-channel and feedback-free policy, in which power is allocated equally to all the carriers of an OFDM signal, thus emerges as a justified choice for practical implementation. Its throughput can be maximized by judicious pilot allocation across blocks which strikes a balance with the channel estimation accuracy.

Future work will address long-term channel variation, and the attendant methods for rate maximization via power control.

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