Joint Power and Rate Control for Packet Coding over Fading Channels

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Abstract

We consider random linear packet coding for fading channels with long propagation delays, such as underwater acoustic channels. We propose a scheme where the number of coded packets to transmit is determined so as to achieve a pre-specified outage/reliability criterion, and investigate joint power and rate control with constrained resources. Using the channel state information which is obtained via feedback from the receiver, the transmitter adjusts its power and the number of coded packets such that the average energy per successfully transmitted bit of information is minimized. Two optimization constraints are imposed: (a) the transmit power should not exceed a maximum level, and (b) the number of coded packets should not exceed a maximum value dictated by the desired throughput and delay. We further extend the results to take into account the effect of inevitable channel estimation errors, and consider the case where the transmitter has only an estimate of the channel gain. We design adaptation policies for such a case based on minimum mean square error (MMSE) channel estimation, taking into account the effect of channel estimation errors in an optimal manner so as to satisfy the required outage/reliability criterion. Finally, we compare the proposed technique to standard automatic repeat request (ARQ) protocols for underwater communications in terms of the throughput efficiency. Analytical results show that substantial energy savings and improvements in throughput efficiency are available from adaptive power/rate control. We also present experimental results obtained using channel gains measured during the Surface Process Acoustic Communication Experiment (SPACE-08), an at-sea underwater experiment conducted off the coast of Martha's Vineyard in the fall of 2008.

I. Introduction

Underwater communication has garnered much interest in recent years with emerging applications in underwater sensor networks, warning systems, off-shore oil and gas platform operations, marine life monitoring, etc. Since electromagnetic waves do not propagate over long distances underwater, acoustic waves remain the preferred choice for a number of applications. The slow speed of sound in water, however, leads to long propagation delays which challenge the efficiency of traditional automatic repeat request (ARQ) techniques such as the stop-and-wait and its selective versions in acoustic communication systems [1].

Random linear packet coding provides an attractive alternative to traditional ARQ techniques for the underwater acoustic channel. Packet coding has the potential to reduce the overall waiting delay, and since it is applied at the packet level (as opposed to bit-level coding), it can be easily implemented in an existing underwater acoustic modem. Packet coding is also particularly well suited for multicast and broadcast scenarios, where it additionally increases the information throughput. In traditional broadcast networks, the receiving nodes lose packets independently and request re-transmission of specific packets from the transmitter individually. A re-transmitted packet thus only benefits the node that requested it. In packet-coded broadcast networks, the re-transmission of coded packets benefits all the requesting nodes at once.

A system employing packet coding buffers a block of M information-bearing packets at the transmitter and encodes them into a larger set of $N \geq M$ coded packets to be transmitted [2]. At the receiver, the original information-bearing packets can be recovered from a subset of any M or more of the received packets. The concept of packet coded networks was first introduced in [3] where it was shown that employing packet coding improves the overall throughput efficiency as compared to traditional networks.

Random linear packet coding for underwater acoustic communication has been studied in [4–8]. Rateless coding for reliable file transfer in underwater acoustic networks was considered in [4] where a feedback link was used to inform the transmitter when to stop sending coded packets. Since the feedback was used less frequently as compared to traditional ARQ techniques, the overall system performance was shown to improve. Optimal broadcasting policy for underwater acoustic networks based on random linear packet coding was investigated in [5], showing performance improvements over traditional ARQ techniques. Optimal schedules for random linear packet coding in half duplex links were investigated in [6], which provided the optimal number of coded packets to minimize the average time (or energy) needed to complete the transmission of a block of information-bearing packets. Random linear packet coding in the absence of a feedback link was introduced in [7] where the number of coded packets to transmit was determined such that the receiver can decode the original packets with a pre-specified reliability.

The work referenced so far addressed a time-invariant channel with a fixed packet erasure rate. Random linear packet coding for a fading channel with time-varying link conditions was considered in [8], where adaptive power/rate control was explored to overcome the effects of channel fading. In [9], standard optimization techniques were used to determine the trade-off between allocation of redundancy to packet-level erasure coding and physical layer channel coding. In [10], an analysis based on differential equations was utilized to analyze the throughput achievable with packet coding, and to design a dynamic power control algorithm that achieved higher multicast throughput. The trade-off between channel coding and ARQ in a Rayleigh block-fading channel was considered in [11]. Performance comparison was made between a heavily coded system which uses fewer re-transmissions and a lightly coded system with more re-transmissions, to show that a lightly coded system increases the system throughput. In [12], the authors employed network coding and adaptive power control to improve network performance in a broadcast cellular network. Cross-layer optimization of physical layer modulation and coding to maximize the system throughput for wireless fading channels was considered in [13]. For channels that experience severe fading, [14] considered the optimization of packet-level and bit-level coding, and concluded that performance is improved by adding more redundancy to erasure correction coding across packets. In [15], the problem of joint optimization of the mean throughput and packet loss rate in network-coded systems was considered. It was found that a feedback-free packet coding approach provided better performance in terms of the mean throughput and packet loss rate. In [16], the problem of joint power and rate control for a block-fading channel was addressed. The authors defined utility and cost function to specify an optimization framework that aims at maximizing the transmission rate while minimizing the power consumption.

In this paper we address the issue of joint power and rate control for an underwater acoustic channel employing random linear packet coding. In our earlier work [8], we described a framework that combines adaptive power control with random linear packet coding. The average energy per bit was chosen as a measure of performance, since underwater systems have limited resources, and conserving energy aims at increasing the system lifetime. We have shown that when employing adaptive power control, there exists an optimal number of coded packets that minimizes the average energy per bit. Similarly, when employing adaptive rate control, there exists an optimal transmit power that minimizes the average energy per bit. In the present work, we aim to perform joint power and rate control with constrained resources. We use a block fading channel model in which the channel gain is decomposed into two parts: the large-scale slowly-varying part which admits feedback, and the small-scale fast-varying part which does not admit feedback and determines the bit error rate performance. A feedback link is used to convey the large-scale channel gain from the receiver to the transmitter, and adjust the transmit power and the number of coded packets. We define two constraints on the available resources: (a) the transmit power cannot exceed a maximum level, and (b) the number of coded packets cannot exceed a maximum value. Under these two constraints, we provide a framework to perform joint power and rate control which aims to minimize the average energy per successfully transmitted bit of information.

In order to implement the adaptation policy, the channel gain needs to be known at the transmitter. However, in practical systems, the channel gain is not known accurately and only its estimate is available at the transmitter. Using an estimate in place of the true value, we introduce a safety margin to develop adaptation policies that ensure a desired outage/reliability. Following [17], we model the large-scale channel as a log-normally distributed process whose dynamics obey a first-order auto regressive (AR-1) process. Assuming a minimum mean squared error (MMSE) estimate of the channel gain, we provide analytical expressions for system performance under channel uncertainty.

Finally, we compare the proposed packet coding technique with traditional ARQ techniques and show performance improvements in terms of throughput efficiency. We also provide experimental results from the Surface Process Acoustic Communication Experiment (SPACE-08), an at-sea underwater experiment conducted off the coast of Martha's Vineyard in the fall of 2008. Using the experimentally recorded channel gains from the SPACE-08 experiment, we show that substantial energy savings and throughput improvements are available using the proposed packet coding technique.

The paper is organized as follows. We present the system model in Sec. II, followed by the optimization criterion and the adaptation policy in Sec. III. The adaptation policy is extended to a system with imperfect channel knowledge in Sec. IV. A comparison of the proposed technique with traditional ARQ techniques is presented in Sec. V. Experimental results from the SPACE-08 experiment are presented in Sec. VI. The conclusions are summarized in Sec. VII.

II. System Model

On a channel with large-scale gain G, the average signal power at the receiver is given by $P_R = GP_T$, where P_T is the transmit power. We assume a block fading model where the channel gain remains constant over a block of packets, but may vary from one block to another. The duration over which the channel gain remains constant is referred to as the coherence time of the channel, and is represented by T_c. The large-scale channel gain is varying slowly, and hence its value can be sent via feedback to the transmitter. In each block, the transmitter buffers a block of M packets and encodes them into $N \geq M$ packets for transmission over the channel. The transmitter employs random linear packet coding to generate the coded data packets. Each original information packet contains N_b bits. A coded packet is generated as a linear combination of the M original data packets. The random coding coefficients that are used to generate the coded packets are appended to the end of each coded packet. Each coding coefficient is represented by q bits when the field over which encoding takes place is GF(2^q) [2]. Thus, the total number of bits in each packet is $C_b = N_b + qM + h$, where h represents any additional overhead, including the header and the bits for cyclic redundancy check (CRC). Each transmitted packet contains K_b bits, C_b of which are from the coded packet and $K_b - C_b$ are channel coding bits. The duration of each packet is $T_p = K_b/R_b$, where R_b is the bit rate in the channel, and the effective information rate is N_b/T_p. After every block of packets, the transmitter waits for a feedback from the receiver which contains the channel gain information.

The signal-to-noise ratio (SNR) at the receiver is given by $\gamma = P_R/P_N = GP_T/P_N$, where P_N is the noise power. The bit error rate (BER) is a function of the SNR and is represented by $P_e(\gamma)$. The corresponding packet error rate is determined by $P_e(\gamma)$, e.g. as $P_E(\gamma) = 1 - (1 - P_e(\gamma)^{C_b}$. Since the channel gain G is randomly varying, so is the packet error rate $P_E(\gamma)$. We note that packet coding does not replace channel coding, but works along with it to improve the throughput efficiency.

The probability of successful decoding, defined as the probability that at least M out of the N coded packets are received correctly is given by

$$P_s(\gamma) = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma))^m P_E^{N-m}(\gamma) \tag{1}$$

We wish to maintain a pre-defined success rate P_s^* at the receiver. We can now define the outage probability as the probability that P_s falls below the pre-defined value P_s^*, i.e.,

$$P_{\text{out}} = \mathbb{P}\{P_s(\gamma) < P_s^*\} \tag{2}$$

We chose the average energy per bit as the figure or merit to determine the optimal transmit power and the number of coded packets. In many underwater applications, power is limited, and hence minimizing the average energy per bit aims at increasing the system lifetime. The average energy per successfully transmitted bit is given by

$$\bar{E}_b = \frac{1}{R_b} \frac{\mathbb{E}\{NP_T\}}{P_s^* M} \frac{C_b}{N_b} \tag{3}$$

The average energy per bit is influenced by both the number of coded packets and the transmit power. On the one hand, increasing the transmit power leads to a higher SNR, and hence fewer coded packets would be necessary, but on the other hand, increasing the transmit power also directly increases the average energy per bit. It is this trade-off that we wish to exploit by finding the optimal values for the transmit power and the number of coded packets.

In a practical system, resources are limited. Keeping this fact in mind, we impose two constraints:

1) The transmit power cannot exceed a maximal level, $P_{T,\text{max.}}$ This level is dictated by the hardware system constraints or by the total budget.
2) The number of coded packets cannot exceed a maximal value, N_{max}. This value is determined so as to satisfy the following requirements: (i) a block must not last longer than a value dictated by the coherence of the channel gain T_c; (ii) the decoding delay T_d must not exceed a maximum tolerable value $T_{d,\text{max}}$, and (iii) the average bit rate must not fall below a tolerable minimum $R_{b,\text{min}}$. N_{max} is thus given as

$$N_{\text{max}} = \min\left\{\frac{T_c}{T_p}, \frac{T_{d,\text{max}}}{T_p}, \frac{N_b P_s^* M}{T_p R_{b,\text{min}}}\right\} \tag{4}$$

III. Optimization procedure

In this section, we define the optimization procedure to determine the transmit power and the number of coded packets so as to maintain a pre-defined reliability while minimizing the average energy per successfully transmitted bit of information. For different values of N starting from M and increasing to $M+1, M+2\ldots$, Fig. 1 shows the relationship between P_s and P_E. As

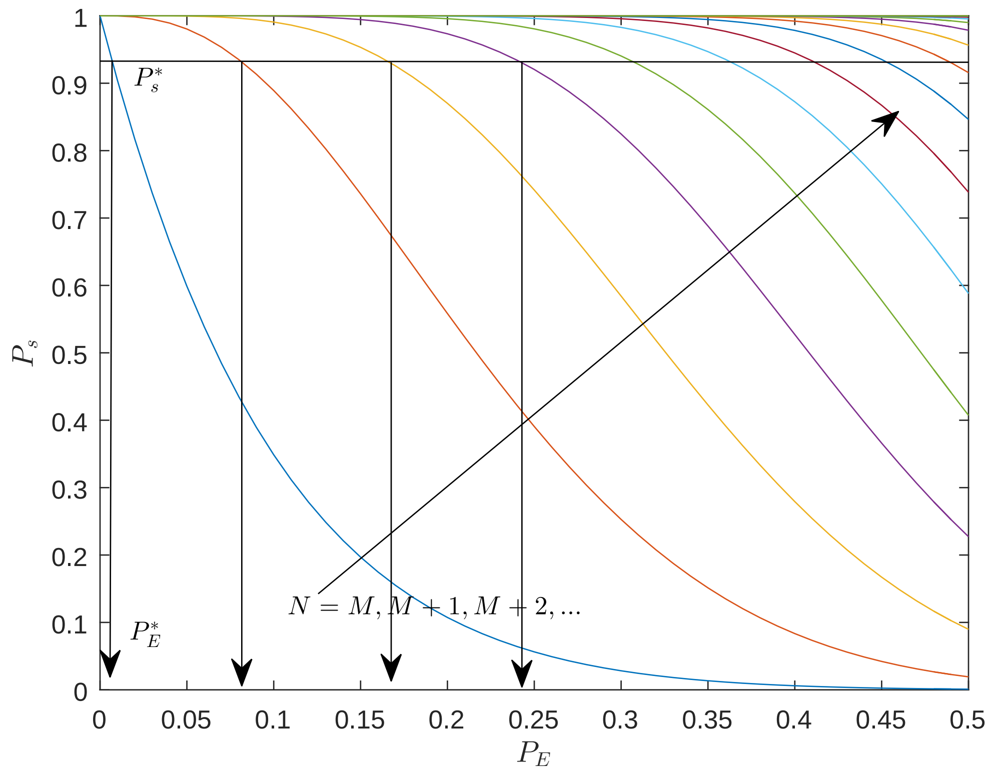


Figure 1: Probability of successful decoding vs. probability of packet error. For a desired P_s^* and a chosen value of N, the figure points to a value of P_E^* which in turn points to a necessary SNR $\gamma^*(N)$.

seen in the figure, for a desired P_s^*, and for every candidate value N, there is a corresponding value P_E^*. Given a specific small-scale fading type and a modulation/coding/diversity scheme, the value P_E^* corresponds to a particular value of the SNR, which we denote as $\gamma^*(N)$.

The relationship between the number of coded packets N and the corresponding $\gamma^*(N)$ is summarized in Fig. 2. Also plotted in Fig. 2 is the product $N\gamma^*(N)$ which is meaningful for minimizing the average energy per bit. For purposes of illustration, we assume Rician fading, and differentially coherent detection with no coding or diversity. Note that our analysis does not change with the change in any of these assumptions; only numerical results do.

For a given channel gain G, the transmit power needed to achieve $\gamma^*(N)$ is

$$P_T^* = \gamma^*(N) P_N / G \tag{5}$$

where P_N is the noise power. Fig. 3 shows the product NP_T^* as a function of N for various

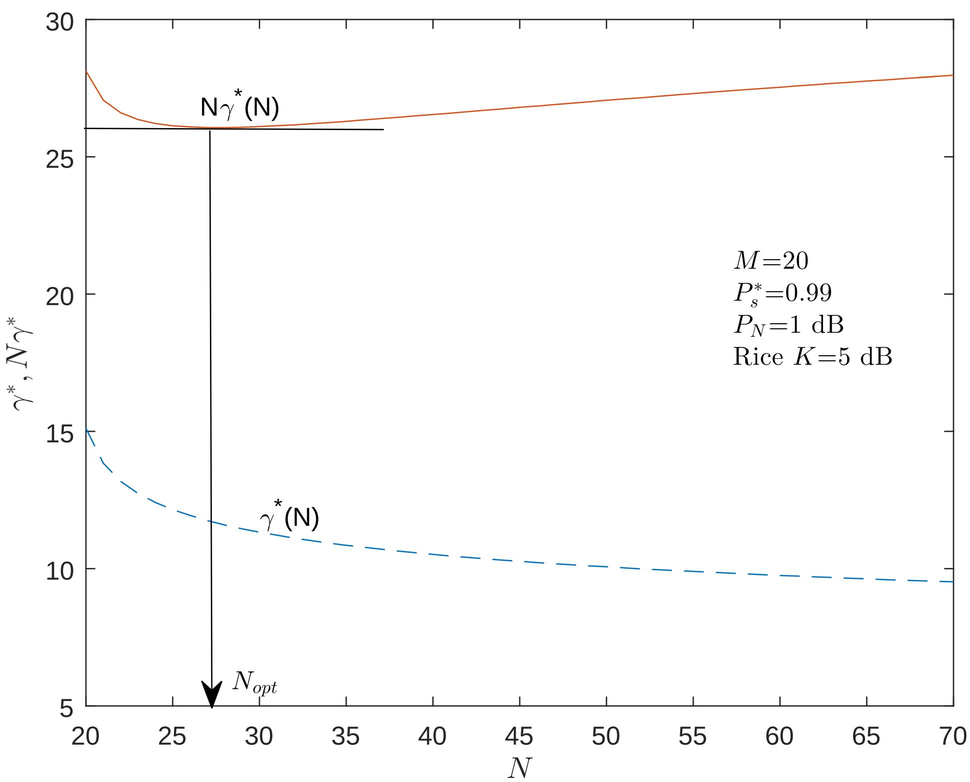


Figure 2: The SNR $\gamma^*(N)$ and the product $N\gamma^*(N)$ as functions of N. The product $N\gamma^*(N)$ is relevant for minimizing the average energy per bit.

values of the gain G. It is important to note that the value of N which minimizes NP_T^* is the same as that which minimizes $N\gamma^*(N)$. Hence, this value does not depend on the channel gain G. We denote this value by N_{opt} as it is the value that minimizes the average energy per bit $\bar{E}_b$ in the absence of any constraints.

In order to accommodate the constraints stated earlier, the optimization procedure is conducted as follows:

1) If the optimal number of coded packets is less than the maximum number of coded packets allowed, i.e., $N_{\text{opt}} \leq N_{\max}$, we choose to transmit $N = N_{\text{opt}}$ coded packets. If $N_{\text{opt}} > N_{\max}$ we choose $N = N_{\max}$.

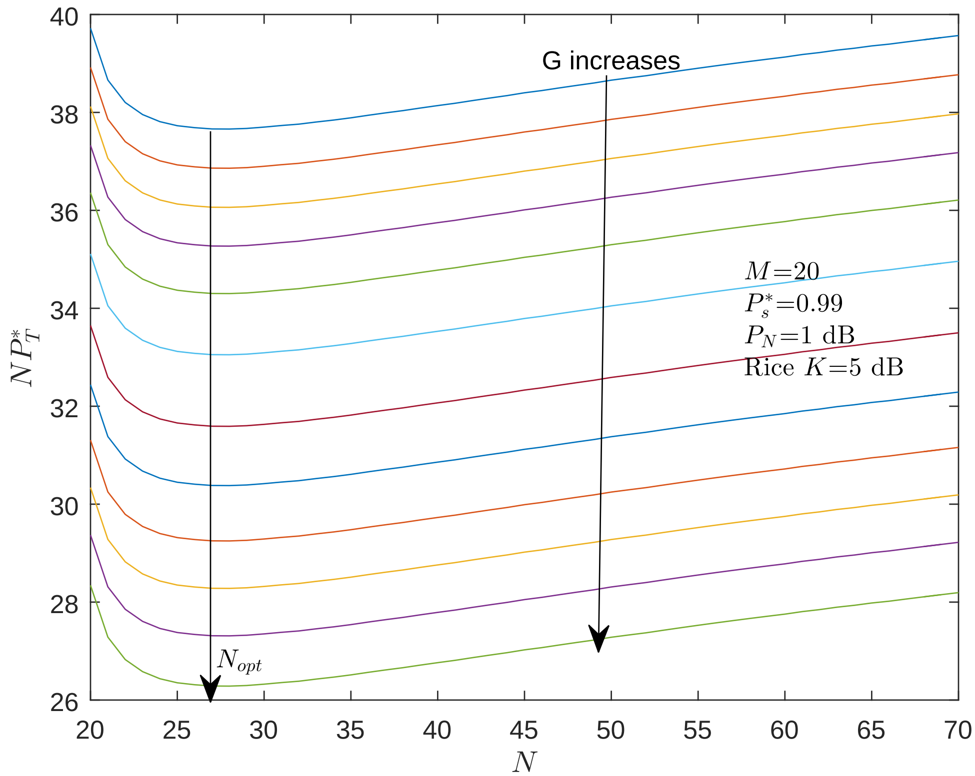


Figure 3: To minimize the average energy per bit, the number of coded packets and the transmit power should be chosen such that the product NP_T^* is minimized (subject to system constraints $P_{T,\max}$, $N_{\max}$).

2) For a given G, we calculate the threshold $\tilde{P}_T = \gamma^*(N)P_N/G$ using the value of N chosen above. If this threshold is below the maximum system power $P_{T,\max}$, we set the transmit power to $P_T = \tilde{P}_T$. Otherwise (i.e. if $\tilde{P}_T > P_{T,\max}$), we have to make a choice based on N that was used to calculate $\tilde{P}_T$, i.e., $N = N_{\max}$ or $N = N_{\text{opt}}$. If $N = N_{\max}$, we have exhausted the system resources, i.e., there is no combination of N and P_T that will satisfy $P_s \geq P_s^*$, and hence we shut the transmission off. If $N = N_{\text{opt}}$, we may increase N above N_{opt} (but not above $N_{\max}$) and see if there exists a value of N for which $\tilde{P}_T$ drops to $P_{T,\max}$. If such a point exists, we choose that combination; otherwise, we shut off.

From the above discussion, it is clear that two cases exist for the system operation. The first case corresponds to the maximum number of coded packets being less than or equal to the optimal number of coded packets, $N_{\max} \leq N_{\text{opt}}$. The second case occurs when $N_{\max} > N_{\text{opt}}$. Below, we analyze each of these cases in detail.

A. *Case 1:* $N_{\max} \leq N_{\text{opt}}$

When $N_{\max} \leq N_{\text{opt}}$, the optimization procedure is straightforward. The number of packets is kept fixed at $N = N_{\max}$, and the power is varied as $P_T = \gamma^*(N_{\max})P_N/G$ if this value is below $P_{T,\max}$; otherwise, transmission is turned off, i.e.,

$$P_T = \begin{cases} \gamma^*(N_{\max})P_N/G, & \text{if } \gamma^*(N_{\max})P_N/G \leq P_{T,\max} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Outage occurs when $P_T G/P_N < \gamma^*(N)$, i.e. when $G < \gamma^*(N_{\max})P_N/P_{T,\max}$ in this case. Under the assumption that the large-scale channel gain can be modeled as log-normally distributed, i.e., $10\log_{10} G \sim \mathcal{N}(\bar{g}, \sigma_g^2)$, the probability of outage is

$$P_{\text{out}} = \mathbb{P}\{G < \underbrace{\gamma^*(N_{\max})P_N/P_{T,\max}}_{G_{\text{out}}}\} = Q\left(\frac{\bar{g} - 10\log_{10} G_{\text{out}}}{\sigma_g}\right) \tag{7}$$

where Q denotes the Q-function, $Q(x) = 1/2\text{erfc}(x/\sqrt{2})$. There are two ways in which the system design can be specified. One can choose to specify the maximum transmit power $P_{T,\max}$, in which case the outage probability follows from the above expression, or by a desired P_{out}, in which case the corresponding value of $P_{T,\max}$ can be obtained from the above expression.

The average energy per bit is

$$\bar{E}_b = \frac{1}{R_b}\frac{N_{\max}\bar{P}_T C_b}{P_s^* M N_b} \tag{8}$$

The average power $\bar{P}_T$ can be computed as

$$\bar{P}_T = \gamma^*(N_{\max})P_N \underbrace{\int_{G_{\text{out}}}^{+\infty} \frac{1}{x} p_G(x)\mathrm{d}x}_{F_G(G_{\text{out}})} \tag{9}$$

where $p_G(\cdot)$ represents the probability density function (p.d.f.) of the channel gain G. The factor $F_G(G_{\text{out}})$ can be computed in closed form (see Appendix A) as

$$F_G(G_{\text{out}}) = 10^{-\bar{g}/10} e^{\left(\frac{\ln 10}{10}\right)^2 \frac{\sigma_g^2}{2}} Q\left(\frac{\ln 10}{10}\sigma_g - Q_{\text{inv}}\left(P_{\text{out}}\right)\right) \tag{10}$$

In order to test the performance of the adaptation scheme, we compare it with a system that operates at a fixed power and rate. If there is no adaptive control, transmit power is kept fixed at some $P_{T,\text{fix}}$, and the number of packets is also fixed at N_{fix}. The SNR, $\gamma = P_{T,\text{fix}}G/P_N$, changes with the gain, and so do the probabilities P_E and P_s. Outage occurs when $\gamma < \gamma^*(N_{\text{fix}})$, and the probability of outage for a log-normally distributed G is

$$P_{\text{out}} = \mathbb{P}\{\gamma < \gamma^*(N_{\text{fix}})\} = \mathbb{P}\{G < \underbrace{\gamma^*(N_{\text{fix}})P_N/P_{T,\text{fix}}}_{G_{\text{fix}}}\} = Q\left(\frac{\bar{g} - 10\log_{10} G_{\text{fix}}}{\sigma_g}\right) \tag{11}$$

The power needed to keep the outage at a pre-specified level P_{out} is thus $P_{T,\text{fix}} = \gamma^*(N_{\text{fix}})P_N/G_{\text{fix}}$, where $10\log_{10} G_{\text{fix}} = \bar{g} - \sigma_g Q^{-1}(P_{\text{out}})$. Compared to a non-fading case ($\sigma_g = 0$), the transmit power is increased by a fixed margin $\sigma_g Q^{-1}(P_{\text{out}})$. For example, if σ_g=10 dB and $P_{\text{out}} = 10\%$, the margin is about 13 dB (for 1% outage, it is 23 dB).

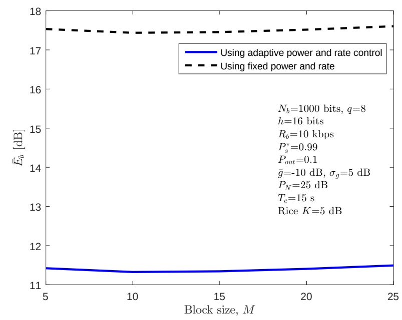
For *Case 1*, the number of coded packets for the fixed scheme is $N_{\text{fix}} = N_{\max}$. The performance of adaptive power and rate control strategies for this case is summarized in Fig. 4. Fig. 4a shows the average energy per bit as a function of the block size M. As seen from the figure, larger block size leads to a higher average energy per bit consumption because of the additional packet coding overhead. One has to keep in mind that the block size cannot become so large that it violates the block fading model. The value of the block size M is thus a design choice made based on the coherence time of the channel. We can see that savings of approximately 6 dB-8 dB are available by employing adaptive power and rate control. Fig. 4b shows the average energy per bit as a function of the packet size (number of information bits) N_b. The packet coding overhead at smaller packet sizes is high, leading to a higher energy per bit consumption. Larger packet sizes are favorable as they lead to a higher average throughput as well as lower average energy per bit. Fig. 4c shows the average energy per bit as a function of the standard deviation σ_g. The savings available from adaptive power and rate control increase with σ_g.

B. *Case 2:* $N_{\max} > N_{\text{opt}}$

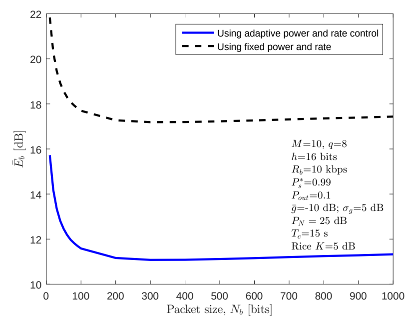
When $N_{max} > N_{\text{opt}}$, the adaptation policy is somewhat more involved. We now have three regions of operation, two in which the system is on, and one in which the transmission is shut off. When the system is on, there are further two modes of operation, one in which the number of packets is kept fixed at $N = N_{\text{opt}}$ and the power is varied, and another in which the number of coded packets N is varied between N_{opt} and $N_{\max}$ the power is kept fixed at $P_T = P_{T,\max}$.

If the gain is high enough such that $\gamma^*(N_{\text{opt}})P_N/G \leq P_{T,\max}$, the number of packets is set to $N = N_{\text{opt}}$, and the power is varied as $P_T = \gamma^*(N_{\text{opt}})P_N/G$. As the gain diminishes, the break point occurs at

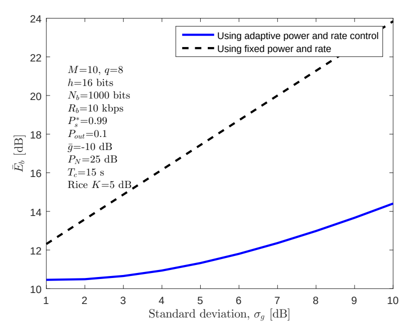
$$G_{\text{break}} = \gamma^*(N_{\text{opt}})P_N/P_{T,\max} \tag{12}$$



(a) Average energy per bit as a function of the block size M. The slight increase in the average energy per bit at larger block sizes is due to the packet coding overhead. It should be noted that the block duration must be kept below the coherence time of the channel.



(b) Average energy per bit as a function of the number of information bits per packet N_b. At very small packet sizes, energy consumption is dominated by the packet coding overhead. Larger packet sizes are favorable as they lead to higher average throughput.



(c) Average energy per bit as a function of the standard deviation σ_g.

Figure 4: Performance figures for *Case 1* ($N_{\max} \leq N_{\text{opt}}$).

If the gain drops below this value, but there exists a value $N \leq N_{\max}$ such that $\gamma^*(N)P_N/G = P_{T,\max}$, that value is chosen, and the power is kept at $P_T = P_{T,\max}$. Rate adaptation is thus performed as $N = \gamma^*_{\text{inv}}(P_{T,\max}G/P_N)$, where the $\gamma^*_{\text{inv}}(\cdot)$ denotes the inverse function of $\gamma^*(\cdot)$. This function follows from the relationship between N and γ^* shown in Fig. 2. The outage point occurs at

$$G_{\text{out}} = \gamma^*(N_{\max})P_N/P_{T,\max} \tag{13}$$

If the gain drops below this value, there is no solution for (N, P_T) that satisfies the required P_s^*, and transmission is shut off.

The probability of outage is again given by the expression (7), but now with $N_{\max} > N_{\text{opt}}$. If the system is designed based on a given $P_{T,\max}$, then this outage probability represents the best that can be achieved. Alternatively, if rate adjustment is not an option, one can settle for operating only with N_{opt}, in which case the outage will be imposed whenever the gain drops below G_{break}. In either case, the design can also be carried "backwards," i.e. starting with a desired P_{out} and determining the necessary $P_{T,\max}$ from it.

The adaptation policy can be visualized in Fig. 5, by following the solid straight lines in the direction of increasing (or decreasing) gain. For instance, starting with a very high gain and going in the direction of decreasing gain, the system follows the vertical line labeled N_{opt}, moving upwards until the point labeled G_{break} is reached. There, a 90° turn is taken to the right, and the system continues to follow the horizontal line labeled $P_{T,\max}$, until the point labeled G_{out} is reached. Thereafter, the system is shut off.

The adaptation policy for the case $N_{\max} > N_{\text{opt}}$ is thus summarized as follows:

$$N = \begin{cases} 0 & \text{if } G < G_{\text{out}} \\ \gamma^*_{\text{inv}}\left(\frac{P_{T,\max}G}{P_N}\right), & \text{if } G \in [G_{\text{out}}, G_{break}] \\ N_{\text{opt}}, & \text{if } G > G_{\text{break}} \end{cases} \tag{14}$$

$$P_T = \begin{cases} 0, & \text{if } G < G_{\text{out}} \\ P_{T,\max}, & \text{if } G \in [G_{\text{out}}, G_{\text{break}}] \\ \frac{\gamma^*(N_{\text{opt}})P_N}{G}, & \text{if } G > G_{\text{break}} \end{cases} \tag{15}$$

The values G_{break} and G_{out} are defined by the expressions (12) and (13).

Fig. 6 summarizes an adaptation policy of this type.

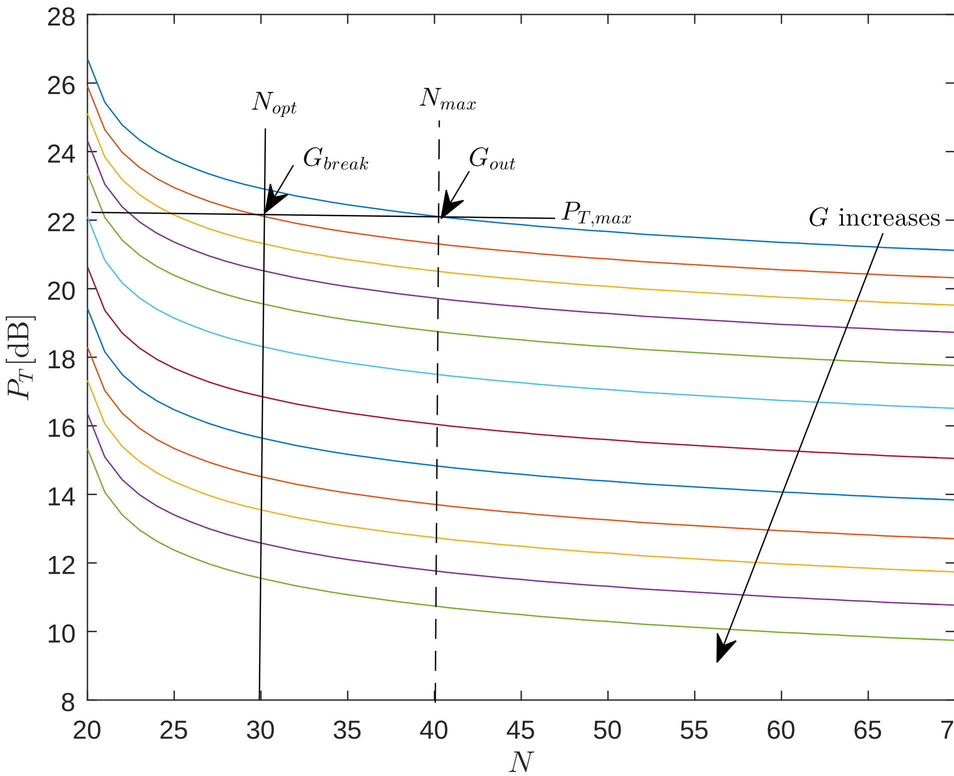


Figure 5: For a given channel gain G, each curve represents the points (N, P_T) that satisfy the outage requirement. For $G < G_{\text{out}}$, there are no such points, and transmission is shut off. In the region between G_{out} and G_{break}, the power is kept fixed at $P_{T,\text{max}}$ and the rate is chosen at the crossing point between the line $P_{T,\text{max}}$ and the given G. The corresponding number of coded packets is between N_{max} and N_{opt}. In the region $G > G_{\text{break}}$, the number of packets is kept fixed at N_{opt}, and the power is chosen at the crossing point between the line N_{opt} and the given G.

The average energy per bit is now given by

$$\bar{E}_b = \frac{1}{R_b}\frac{C_b}{P_s^* M N_b}\left[P_{T,\max}\int_{G_{\text{out}}}^{G_{\text{break}}}\gamma_{\text{inv}}^*\left(\frac{xP_{T,\max}}{P_N}\right)p_G(x)\mathrm{d}x + N_{\text{opt}}\gamma^*(N_{\text{opt}})P_N\underbrace{\int_{G_{\text{break}}}^{+\infty}\frac{1}{x}p_G(x)\mathrm{d}x}_{F_G(G_{\text{break}})}\right] \tag{16}$$

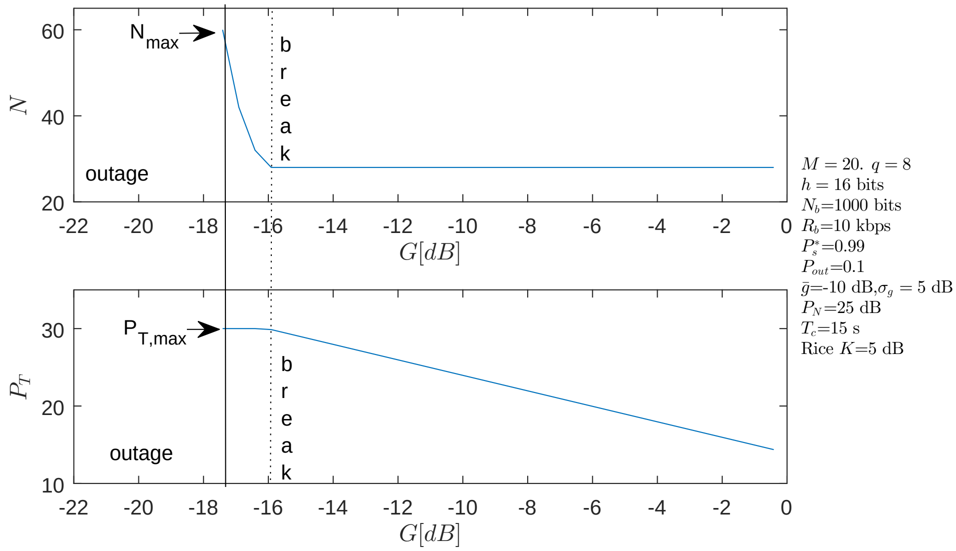
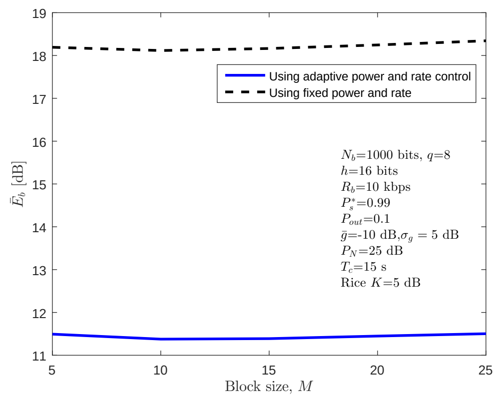


Figure 6: Rate (top) and power (bottom) control policy for the case when $N_{\text{opt}} < N_{\max}$.

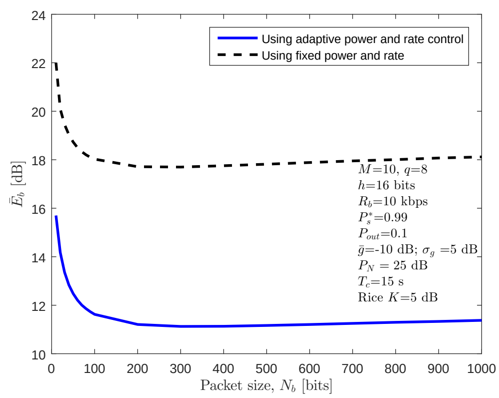
The system performance for $N_{\max} > N_{\text{opt}}$ is shown in Fig. 7. For the benchmark case with fixed power and rate, we choose $N_{\text{fix}} = N_{\text{opt}}$. As before, we plot the energy per bit as a function of the block size (Fig. 7a), the packet size (Fig. 7b), and the standard deviation σ_g of the channel gain (Fig. 7c). We observe a similar trend as in *Case 1*, although a slightly higher energy savings are available in this case since the transmitter has an additional degree of freedom (number of coded packets).

IV. Adaptive power and rate control in the presence of channel estimation errors

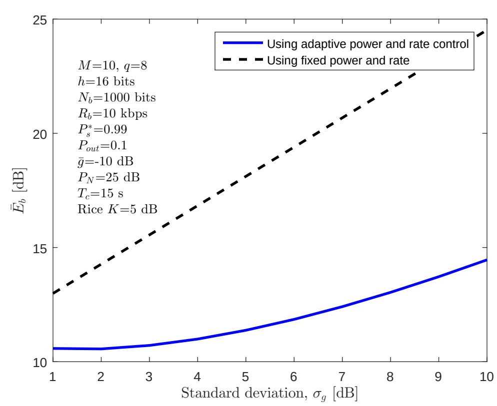
Our analysis so far was based on the assumption that perfect channel state information is available to the transmitter. However, in practice, due to the noisy estimation carried out at the receiver, and the feedback delay, the transmitter only has an estimate of the channel gain. The discrepancy between the estimate $\hat{G}$ and the true channel gain G has to be taken into account to meet a desired outage criterion. Specifically, this discrepancy will lead to additional outage events, which, together with the outage due to shut-off, now contribute to the overall outage rate, and the system has to be designed such that the overall outage remains at the design value. In order to keep the overall outage at the design value, we introduce an additional margin C to control the outage due to estimation errors. With the inclusion of the margin, we formulate the



(a) Average energy per bit as a function of the block size M. As in *Case 1*, there is a slight increase in the average energy per bit at larger block sizes, which is caused by the packet coding overhead.



(b) Average energy per bit as a function of the number of information bits per packet N_b.



(c) Average energy per bit as a function of the standard deviation σ_g. As seen in *Case 1,* higher energy savings are available from packet coding when σ_g increases.

Figure 7: Performance figures for *Case 2* ($N_{\max} > N_{\text{opt}}$).

adaptive power and rate control policy for each of the cases discussed earlier, now utilizing the estimated channel gain.

A. Case 1: $N_{\max} \leq N_{\text{opt}}$, with imperfect channel knowledge

For this case, we fix the number of coded packets at $N_{\max}$ and adapt the power as

$$P_T = \begin{cases} C\gamma^*(N_{\max})P_N/\hat{G}, & \text{if } C\gamma^*(N_{\max})P_N/\hat{G} \leq P_{T,\max} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

This is the same policy as before, except that the gain is replaced by its estimate, and the margin C is introduced. The margin provides an additional degree of freedom needed to meet the outage requirement.

The probability of outage is now dictated by two factors: outage when the system is turned off, and outage that occurs when the system is turned on but we have an imperfect channel information. In other words,

$$P_{\text{out}} = P_{\text{off}} + P_{\text{on}}P_{\text{out}|\text{on}} \tag{18}$$

where $P_{\text{off}} = 1 - P_{\text{on}}$ is the probability that transmission is shut off,

$$P_{\text{off}} = \mathbb{P}\{C\gamma^*(N_{\max})P_N/\hat{G} > P_{T,\max}\} = \mathbb{P}\{\hat{G} < C\underbrace{\gamma^*(N_{\max})P_N/P_{T,\max}}_{G_{\text{out}}}\} \tag{19}$$

and

$$P_{\text{out}|\text{on}} = \mathbb{P}\{CG/\hat{G} < 1 \mid \hat{G} \geq CG_{\text{out}}\} \tag{20}$$

is the probability of outage when transmission is active.

For the log-normally distributed gain, we assume that MMSE estimation is employed on the dB scale, i.e. that the gain $g = \bar{g} + \Delta g$ is estimated as $\hat{g} = \bar{g} + \Delta\hat{g}$, such that the estimation error $e = \Delta g - \Delta\hat{g}$ is orthogonal to the estimate $\Delta\hat{g}$. In that case, the outage probability reduces to

$$P_{\text{out}} = \mathbb{P}\{\Delta\hat{g} < c+g_{\text{out}}-\bar{g}\}+(1-\mathbb{P}\{\Delta\hat{g} < c+g_{\text{out}}-\bar{g}\})\underbrace{\mathbb{P}\{e_g < -c \mid \Delta\hat{g} > c+g_{\text{out}}-\bar{g}\}}_{\mathbb{P}\{e_g > c\}} \tag{21}$$

where $c = 10\log_{10} C$, $g_{\text{out}} = 10\log_{10} G_{\text{out}}$, and we use the fact that the zero-mean Gaussian error e_g is orthogonal to the MMSE estimate $\Delta\hat{g}$. Expressing the constituent probabilities in terms of the Q-function, we obtain

$$P_{\text{out}} = Q\left(\frac{\bar{g} - g_{\text{out}} - c}{\sigma_{\hat{g}}}\right) + \left[1 - Q\left(\frac{\bar{g} - g_{\text{out}} - c}{\sigma_{\hat{g}}}\right)\right] Q\left(\frac{c}{\sigma_e}\right) \tag{22}$$

where $\sigma_{\hat{g}}^2 = E\{\Delta\hat{g}^2\}$ and $\sigma_e^2 = E\{e_g^2\}$.

Based on [17], we further assume that Δg obeys an auto-regressive Gauss-Markov process of order 1, described by the one-step correlation coefficient a. This model applies to a class of shallow water channels for which it was validated experimentally. The details of the model can be found in [17]. Following this,

$$\Delta g[n] = a\Delta g[n-1] + w_g[n] \tag{23}$$

$$g[n] = \bar{g} + \Delta g[n] \tag{24}$$

where the clock n ticks with each block, i.e., one feedback cycle of duration T_s, and $w_g[n]$ represents the process noise which is Gaussian, zero-mean and independent of $\Delta g[n-1]$. Assuming a stationary process with known statistics, the estimate of the gain is given by

$$\Delta\hat{g}[n] = a\Delta g[n-1] \tag{25}$$

$$\hat{g}[n] = \bar{g} + \Delta\hat{g}[n] \tag{26}$$

In this case, we have that $\sigma_{\hat{g}}^2 = a^2\sigma_g^2$, and $\sigma_e^2 = (1-a^2)\sigma_g^2$. Both variances are thus specified through the parameter a, which is in turn related to the Doppler bandwidth B_g of the process $g[n]$ via $a = e^{-\pi B_g T_s}$. Substituting for the variance, the outage probability can now be expressed as

$$P_{\text{out}} = Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right) + \left[1 - Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right)\right] Q\left(\frac{c}{\sqrt{1-a^2}\sigma_g}\right) \tag{27}$$

Given the statistics of the log-normal slow fading (the mean and variance $\bar{g}, \sigma_g^2$, and the correlation factor a) the above expression can be used to determine the margin that will meet a desired outage requirement. Fig. 8 shows the probability P_{out} as a function of the margin C. As the correlation a increases, the channel becomes more predictable, and hence a lower C suffices to reach a desired P_{out}.

The average energy per bit is given by

$$\bar{E}_b = \frac{1}{R_b}\frac{N_{\max}C_b}{P_s^* M N_b} C\gamma^*(N_{\max}) P_N \underbrace{\int_{CG_{\text{out}}}^{+\infty} \frac{1}{x} p_{\hat{G}}(x)\mathrm{d}x}_{F_{\hat{G}}(CG_{\text{out}})} \tag{28}$$

where $p_{\hat{G}}(\cdot)$ denotes the p.d.f. of the estimate $\hat{G}$.

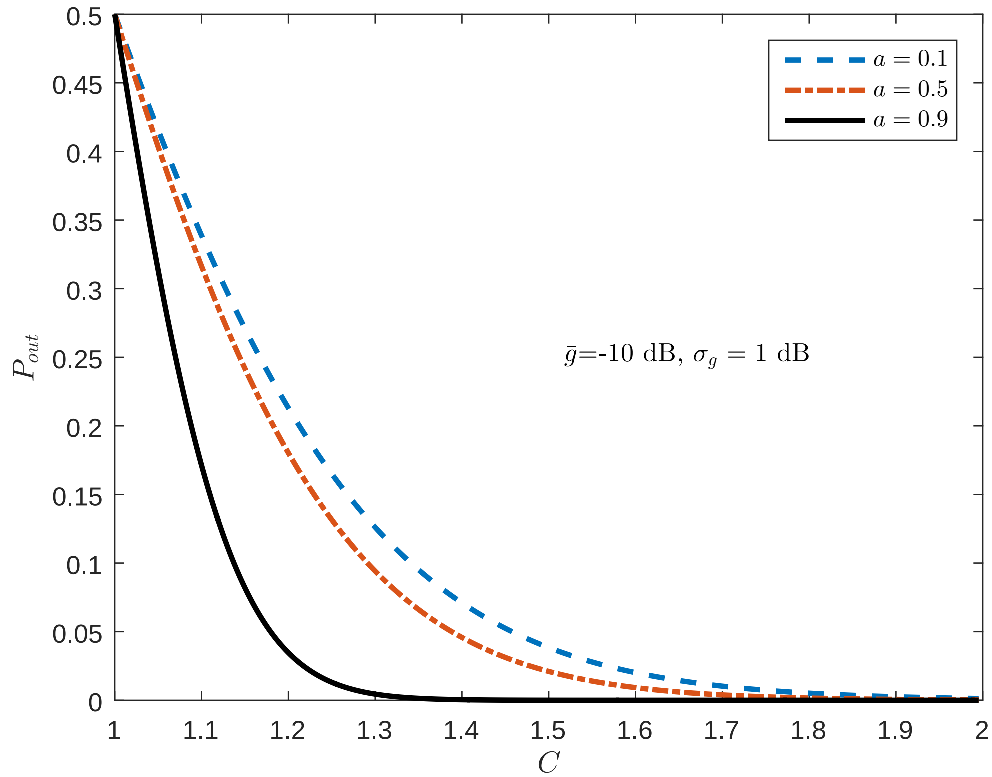
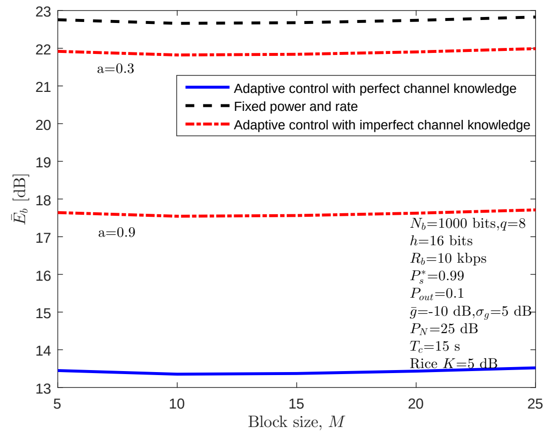
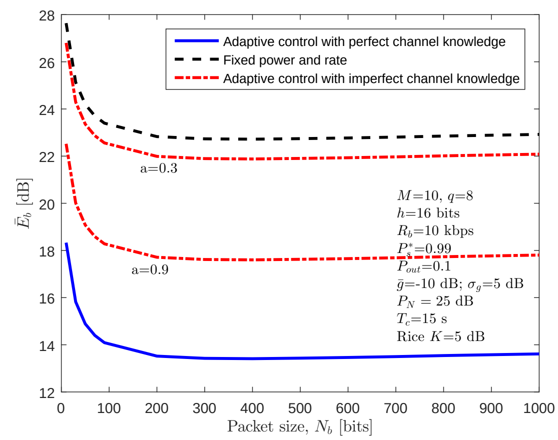


Figure 8: P_{out} as a function of the margin C. A design value of P_{out} implies the necessary margin C for a given set of statistical parameters $\bar{g}$, σ_g and a.

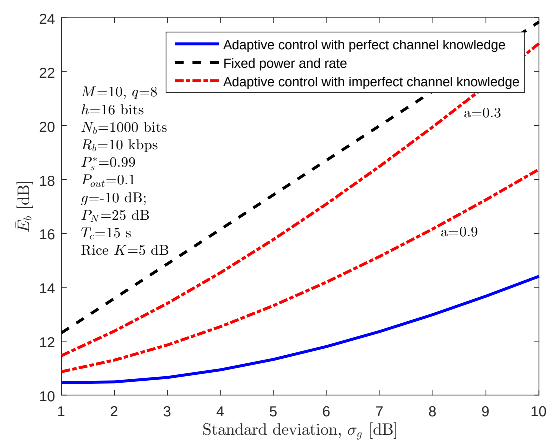
Fig. 9 shows the performance for *Case 1* with imperfect channel knowledge. Different performance curves correspond to different values of the correlation factor a. It should be noted that $a = 1$ corresponds to the ideal case of perfect channel knowledge.



(a) Average energy per bit as a function of the block size M.



(b) Average energy per bit as a function of the number of information bits per packet N_b.



(c) Average energy per bit as a function of the standard deviation σ_g.

Figure 9: Performance figures for *Case 1* with imperfect channel knowledge. Results are shown for two different values of the correlation coefficient a (0.3 and 0.9).

B. Case 2: $N_{\max} > N_{\text{opt}}$ with imperfect channel knowledge

In this case, since we adapt both power and rate in accordance with the channel gain, we define the adaptation policy as

$$N = \begin{cases} 0 & \text{if } \hat{G} < CG_{\text{out}} \\ \gamma^*_{\text{inv}}\left(\frac{P_{T,\max}\hat{G}}{CP_N}\right), & \text{if } \hat{G} \in [CG_{\text{out}}, CG_{\text{break}}] \\ N_{\text{opt}}, & \text{if } \hat{G} > CG_{\text{break}} \end{cases} \tag{29}$$

$$P_T = \begin{cases} 0, & \text{if } \hat{G} < CG_{\text{out}} \\ P_{T,\max}, & \text{if } \hat{G} \in [CG_{\text{out}}, CG_{\text{break}}] \\ \frac{C\gamma^*(N_{\text{opt}})P_N}{\hat{G}}, & \text{if } \hat{G} > CG_{\text{break}} \end{cases} \tag{30}$$

where $G_{\text{out}} = \gamma^*(N_{\max})P_N/P_{T,\max}$ and $G_{\text{break}} = \gamma^*(N_{\text{opt}})P_N/P_{T,\max}$ as before.

There are again three regions of operation, one in which the system is shut off, the other two in which the system is on. We define r_1 as the event where $\hat{G} \in [G_{\text{out}}, G_{\text{break}}]$ and r_2 as the event where $\hat{G} > G_{\text{break}}$. When the system is on, the system can operate in conditions of either r_1 or r_2. The outage probability can thus be written as

$$P_{\text{out}} = P_{\text{off}} + P_{r_1}P_{\text{out}|r_1} + P_{r_2}P_{\text{out}|r_2} \tag{31}$$

where

$$P_{\text{off}} = \mathbb{P}\{C\gamma^*(N_{\max})P_N/\hat{G} > P_{T,\max}\} = \mathbb{P}\{\hat{G} < CG_{\text{out}}\} \tag{32}$$

The probabilities of outage conditioned on r_1 and r_2 are

$$P_{\text{out}|r_1} = \mathbb{P}\{CG/\hat{G} < 1 \mid \hat{G} \in [G_{\text{out}}, G_{\text{break}}]\} \tag{33}$$

which reduces to

$$P_{\text{out}|r_1} = \mathbb{P}\{e_g < -c \mid \Delta\hat{g} \in [c + g_{\text{break}} - \bar{g}\,,\, c + g_{\text{out}} - \bar{g}\} = \mathbb{P}\{e_g > c\} \tag{34}$$

and,

$$P_{\text{out}|r_2} = \mathbb{P}\{CG/\hat{G} < 1 \mid \hat{G} \geq CG_{\text{break}}\} = \underbrace{\mathbb{P}\{e_g < -c \mid \Delta\hat{g} > c + g_{\text{break}} - \bar{g}\}}_{\mathbb{P}\{e_g > c\}} \tag{35}$$

Expressing the constituent probabilities in terms of the $Q-$function for the case of log-normal variation, and assuming that Δg obeys an auto-regressive Gauss-Markov process of order 1, we obtain

$$P_{\text{out}} = Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right) + \left[1 - Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right)\right] Q\left(\frac{c}{\sqrt{1 - a^2}\sigma_g}\right) \tag{36}$$

The average energy per bit is now given as

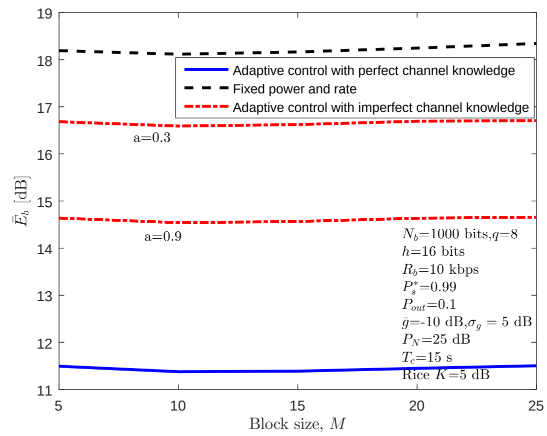
$$\bar{E}_b = \frac{1}{R_b}\frac{C_b}{P_s^* M N_b}\left[P_{T,\max}\int_{CG_{\text{out}}}^{CG_{\text{break}}} \gamma_{\text{inv}}^*\left(\frac{xP_{T,\max}}{CP_N}\right) p_{\hat{G}}(x)\mathrm{d}x + N_{\text{opt}} C\gamma^*(N_{\text{opt}})P_N \underbrace{\int_{CG_{\text{break}}}^{+\infty} \frac{1}{x} p_{\hat{G}}(x)\mathrm{d}x}_{F_{\hat{G}}(CG_{\text{break}})}\right] \tag{37}$$

Fig. 10 shows the performance for *Case 2* with imperfect channel information. The results show a trend similar to *Case 1*, although slightly higher energy savings is available since the transmitter can now adapt the rate in addition to adapting the power.

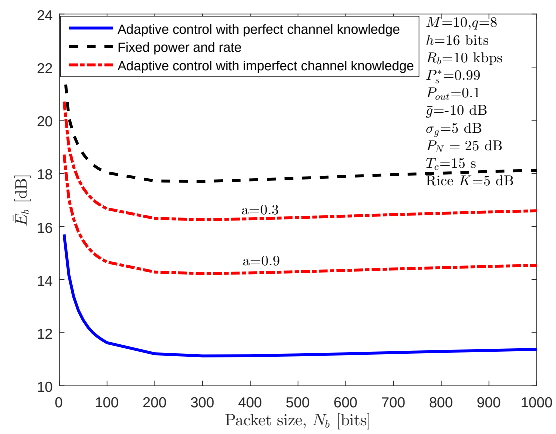
V. Comparison with other ARQ techniques

In this section, we compare the proposed packet coding technique with traditional ARQ techniques. Given the half-duplex operation of acoustic modems, we are limited to the Stop and Wait (S&W) family of ARQ protocols. We consider the basic S&W protocol, as well as its modifications proposed in [1] to optimize the throughput over long-delay acoustic channels.

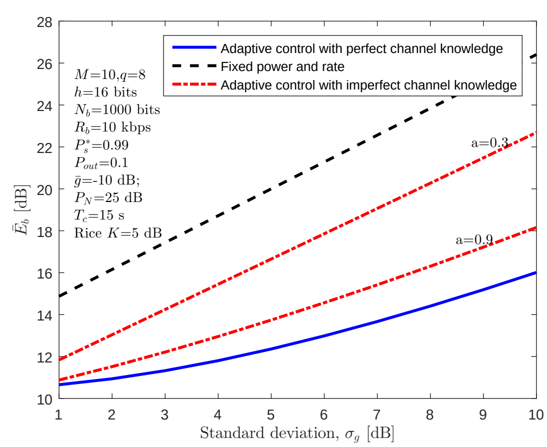
The simplest ARQ protocol is the basic S&W which we call S&W-1, where the transmitter sends a packet and waits for the acknowledgment (ACK). If the ACK is not received within a pre-specified amount of time (time-out), or a negative ACK is received, the packet is re-transmitted. In the modified version S&W-2 [1], the transmitter sends a group of packets, say M, and waits for the acknowledgment. The receiver checks individual received packets and sends acknowledgments at the end of M packets. The packets that are negatively acknowledged are grouped together with new packets to form the next group of M packets. In the modified version S&W-3, the transmitter sends out a group of M packets and waits for the acknowledgment, but only those packets that are negatively acknowledged are transmitted in the next cycle. The transmitter keeps attempting re-transmission until all the M packets are correctly acknowledged. We only consider S&W-2 in this comparison as it outperforms S&W-3 in terms of throughput efficiency.



(a) Average energy per bit as a function of the block size M.



(b) Average energy per bit as a function of the number of information bits per packet N_b.



(c) Average energy per bit as a function of the standard deviation σ_g.

Figure 10: Performance figures for *Case 2* with imperfect channel knowledge.

The propagation delay introduced by the communication channel is $T_{\text{prop}} = d/c$, where d is the distance between the transmitter and receiver, and c is the speed of sound in water (nominally 1500 m/s). The duration of the ACK packet is $T_{\text{ack}} = N_{\text{ack}}/R_b$, where N_{ack} is the number of bits in the ACK packet. Although each of the techniques may have a slightly different ACK duration, we assume the same ACK duration, since it is negligible in comparison to the packet duration. We define the round-trip waiting time as $T_w = 2T_{\text{prop}} + T_{\text{oh}}$, where T_{oh} is any overhead time that includes the time needed for synchronization.

A. Throughput efficiency of packet coding

A system employing packet coding buffers a block of M packets and encodes them into $N \geq M$ packets. The time taken to transmit N packets is

$$T_{PC}(N) = NT_p + T_{\text{ack}} + T_w \tag{38}$$

The efficiency is given by the ratio of useful time to the total time invested in transmission,

$$\eta_{PC} = \frac{P_s^* M(N_b/R_b)}{T_{PC}(N)} \tag{39}$$

This efficiency corresponds to a certain reliability/outage specification, i.e., to a certain P_s^* and P_{out}. When joint power and rate control is employed, we have two cases as described earlier. For *Case 1*, i.e., $N_{\max} \leq N_{\text{opt}}$, the number of packets is fixed at $N_{\max}$, and hence the time taken to transmit is $T_{PC}(N_{\max})$. The corresponding efficiency is

$$\eta_{PC,1} = \frac{P_s^* M(N_b/R_b)}{T_{PC}(N_{\max})} \tag{40}$$

For *Case 2*, i.e., $N_{\max} > N_{\text{opt}}$, under the assumption that perfect channel knowledge is available, the average number of coded packets is given by

$$\bar{N} = \int_{G_{\text{out}}}^{G_{\text{break}}} \gamma_{\text{inv}}^*(xP_{T,\max}/P_N)p_G(x)\mathrm{d}x + N_{\text{opt}} \int_{G_{\text{break}}}^{+\infty} p_G(x)\mathrm{d}x \tag{41}$$

The corresponding efficiency is given by

$$\eta_{PC,2} = \frac{P_s^* M(N_b/R_b)}{T_{PC}(\bar{N})} \tag{42}$$

If perfect channel knowledge is not available, an estimate of the channel gain will be used along with the additional margin as discussed in Sec. IV. For simplicity, we consider only the case of perfect channel knowledge for the present comparison as it suffices to illustrate the point, and yields results similar to those that take imperfect channel knowledge into account.

B. Throughput efficiency of S&W-1

Theoretically, S&W techniques are designed to provide reliable transmission, i.e. a success rate of $P_s = 1$. Since this would entail a possibility of infinitely many re-transmissions, a time-out mechanism is used in practice to limit the number of re-transmissions to some maximum value L^*. As a result, a re-transmission may fail, effectively leading to outage. In order to make a fair comparison between an ARQ technique and the packet coding technique that has a reliability of P_s^*, we choose L^* such that the S&W-1 technique has the same reliability.

We assume that the S&W-1 transmitter has the same $P_{T,\max}$, and hence the best achievable SNR when the gain is G_{out}, is $\gamma^*_{ARQ} = P_{T,\max} G_{\text{out}}/P_N$. Power control for the ARQ system is implemented as

$$P_T = \begin{cases} \gamma^*_{ARQ} P_N/G, & \gamma^*_{ARQ} P_N/G \leq P_{T,\max} \\ 0, & \text{otherwise} \end{cases} \tag{43}$$

The packet error rate is now given by $P_E(\gamma^*_{ARQ})$ and the reliability of the S&W-1 protocol can be defined as the probability that at least one transmission is successful out of the L re-transmission attempts, i.e.,

$$P_{s,SW1} = \sum_{\ell=1}^{L} \binom{L}{\ell} (1 - P_E(\gamma^*_{ARQ}))^{\ell} P_E^{L-\ell}(\gamma^*_{ARQ}) \tag{44}$$

The maximal number of re-transmissions L^* is now obtained as the smallest L for which $P_{s,SW1} \geq P_s^*$.

The average time taken to transmit a packet is

$$T_{SW1} = \sum_{\ell=1}^{L^*} P_E^{\ell-1}(\gamma^*_{ARQ})(1 - P_E(\gamma^*_{ARQ}))\ell T_{ARQ}(1) \tag{45}$$

where $T_{ARQ}(1) = \left(\frac{K_b}{R_b}\right) + T_{\text{ack}} + T_w$ and K_b is the total number of bits in a packet (now without the packet coding overhead). The above reduces to

$$T_{SW1} = \left(\frac{1 - P_E^{L^*}(\gamma^*_{ARQ})}{1 - P_E(\gamma^*_{ARQ})} - L^* P_E^{L^*}(\gamma^*_{ARQ}) \right) T_{ARQ}(1) \tag{46}$$

The corresponding throughput efficiency is

$$\eta_{SW1} = \frac{P_s^* \frac{N_b}{R_b}}{T_{SW1}} \tag{47}$$

C. Throughput efficiency of S&W-2

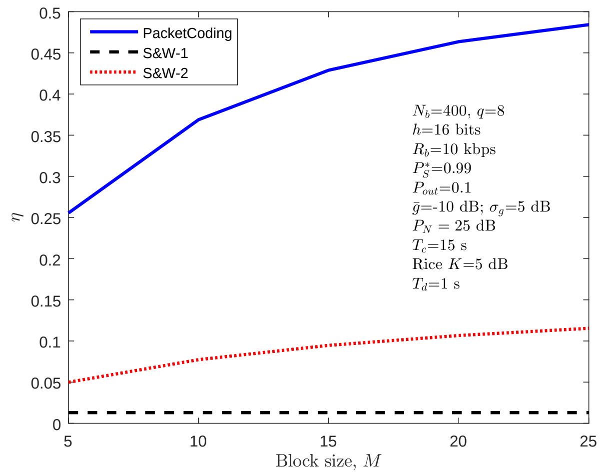
The S&W-2 protocol can be regarded as M S&W-1 protocols operating in parallel with the average time taken to transmit a packet on one of the M links given by

$$T_{SW2} = \left(\frac{1 - P_E^{L^*}(\gamma^*_{ARQ})}{1 - P_E(\gamma^*_{ARQ})} - L^* P_E^{L^*}(\gamma^*_{ARQ}) \right) T_{ARQ}(M) \tag{48}$$

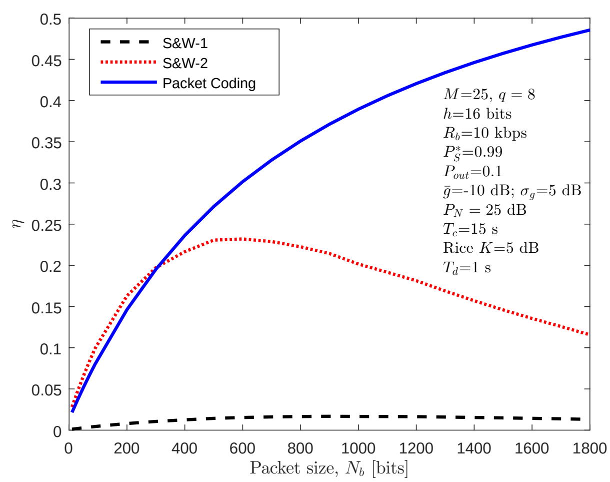
where $T_{ARQ}(M) = M((K_b/R_b) + T_{\text{ack}}) + T_w$. The corresponding efficiency is given as

$$\eta_{SW2} = \frac{P_s^* M \frac{N_b}{R_b}}{T_{SW2}} \tag{49}$$

Fig. 11 shows the throughput of various techniques as a function of the block size M and the packet size N_b. These results show that the packet coding technique outperforms the S&W techniques while maintaining the same reliability. Particularly, for larger packets, the efficiency of the S&W techniques reduces, while packet coding combined with adaptive power and rate control continues to operate with acceptable throughput efficiency.



(a) Throughput efficiency as a function of the block size M. The efficiency of S&W-1 is included as a benchmark (it does not depend on M). Although longer blocks are favorable for increased efficiency, it should be noted that they require larger buffers to store the M packets until they are positively acknowledged.

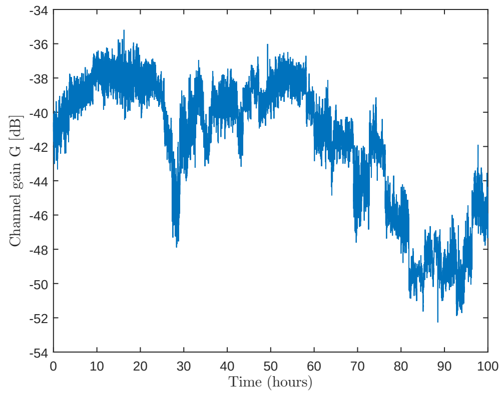


(b) Throughput efficiency as a function of the number of information bits in a packet N_b.

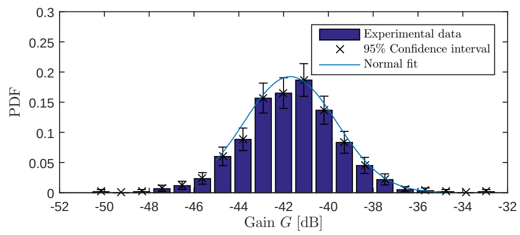
Figure 11: Throughput efficiency.

VI. Experimental results

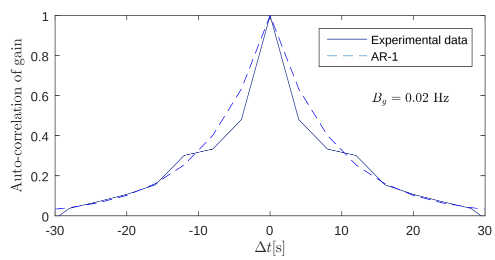
In order to quantify the performance of adaptive power and rate control on an acoustic channel, we use the experimentally recorded values of the channel gain from the Surface Processes



(a) Channel gain recorded during the SPACE-08 experiment.



(b) Histogram of the locally averaged gain (dB scale). The averaging window is 10 s.



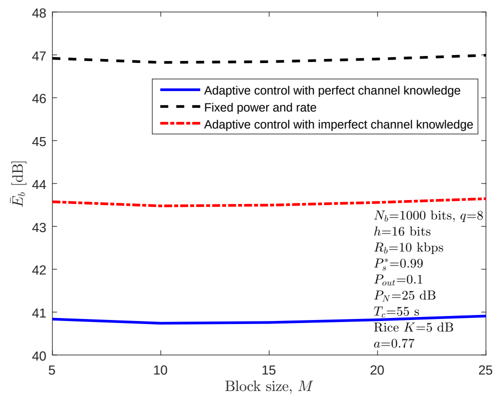
(c) Auto-correlation of the channel gain (dB scale). The channel remains stable over several seconds.

Figure 12: SPACE-08 experiment data.

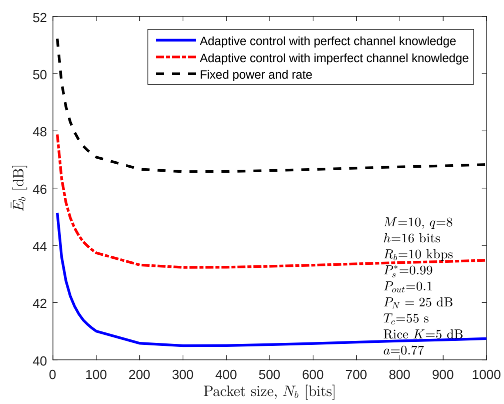
Acoustic Communication Experiment (SPACE) conducted off the coast of Martha's Vineyard in the fall of 2008. In this experiment, a pseudo-random channel probing sequence of length 4095 was transmitted repeatedly, modulated using binary phase shift keying (BPSK) onto a carrier of frequency of 12.5 kHz. The transmitter and receiver were separated by a distance of 1 km. The ocean depth in the region was 10 m and the transmitter and receiver were fixed at a height of 2 m and 4 m from the ocean floor, respectively.

The average channel gain measured from the data is $\bar{g} = -40$ dB and the channel gain standard deviation is $\sigma_g = 3.9$ dB. The 3 dB Doppler bandwidth of the fitted AR-1 model is estimated to be $B_g = 0.02$ Hz, corresponding to the channel correlation coefficient of $a = 0.77$ for $T_s = 4.5$ s. The experimentally recorded gain values are shown in Fig. 12a along with its histogram (Fig. 12b) and auto-correlation function (Fig. 12c). We note that these values are characteristic of the particular SPACE'08 experiment, while other environments may exhibit different statistics. For instance, [17] reports on coherence times $T_c \sim 1/B_g$ of 180 s and 250 s observed in locations near Rhode Island and Hawaii, respectively. These coherence times are considerably longer than in the SPACE'08 experiment, thus allowing for more accurate estimation of the channel gain.

The measured channel statistics were used to assess the performance of power/rate control. Fig. 13 shows the results obtained for *Case 1*, ($N_{\max} \leq N_{\text{opt}}$). We can see the average energy savings of about 6 dB when the channel gain is perfectly known and about 4 dB with the estimated channel.



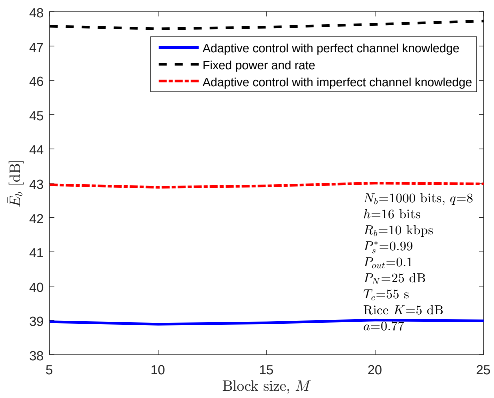
(a) Average energy per bit as a function of the block size M.



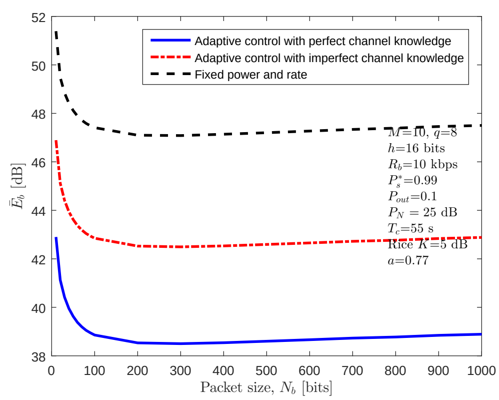
(b) Average energy per bit as a function of the number of information bits per packet N_b.

Figure 13: Average energy per bit calculated for *Case 1* using the experimentally measured channel statistics.

Fig. 14 shows the experimental results for *Case 2*, ($N_{\max} > N_{\text{opt}}$). As with the analytical results, compared with *Case 1,* slightly higher average energy savings are available by using adaptive power and rate control (about 9 dB with perfect channel knowledge, and about 6 dB with estimated channel).



(a) Average energy per bit as a function of the block size M.



(b) Average energy per bit as a function of the number of information bits per packet N_b.

Figure 14: Average energy per bit calculated for *Case 2* using the experimentally measured channel statistics.

VII. Conclusion

We considered random linear packet coding as an alternative to traditional ARQ techniques whose efficiency is compromised by the long round-trip delay of acoustic channels. We proposed a system design in which the number of coded packets to transmit is determined based on pre-specified reliability, hence reducing the need for frequent feedbacks.

Most underwater deployments are offshore and hence have limited power supply for their operation. In such scenarios it becomes essential to prolong the system's lifetime by saving the available energy. To that end, our system employs power and rate control which are optimized so as to minimize the average energy invested per successfully transmitted bit of information. It does so within the constraints on maximal transmit power and maximal coding length (the latter follows from the acceptable decoding delay and bit rate, as well as channel dynamics). The existence of optimal coding length was established, giving rise to two control policies, depending on whether the optimal packet coding length is below or above the allowable maximum. The control policies were further extended to accommodate practical situations in which perfect knowledge of the channel conditions is not available at the transmitter, and an estimate has to be used instead. We analyzed such a case based on MMSE channel estimation, taking into account the effect of channel estimation errors by introducing an optimally designed margin so as to satisfy the required outage probability. We showed both analytically and using experimentally measured channel statistics that savings of $5-9$ dB are achievable using the adaptive power/rate control. Finally, the proposed technique was compared to traditional ARQ techniques to show that employing packet coding with joint power and rate control increases the throughput efficiency.

Future research will focus on extending the joint power and rate control to broadcast networks and relay links. We will also explore random linear packet coding for situations in which full reliability is required.

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Appendix A

Given a log-normally distributed random variable G, let $G = e^A = 10^{B/10}$. Thus,

$$A = \ln G,\ \ B = 10\log_{10} G,\ \ A = \frac{\ln 10}{10}B \tag{50}$$

and the corresponding mean values and variances satisfy

$$m_A = \frac{\ln 10}{10}m_B,\ \ \sigma_A^2 = (\frac{\ln 10}{10})^2\sigma_B^2 \tag{51}$$

We now have

$$F_G(G_{\text{out}}) = \int_{G_{\text{out}}}^{+\infty}\frac{1}{x}p_G(x)\mathrm{d}x = \int_{A_{\text{out}}}^{+\infty}e^{-a}p_A(a)\mathrm{d}a = \int_{e^{A_{\text{out}}}}^{+\infty}e^{-a}\frac{1}{\sqrt{2\pi}\sigma_A}e^{-\frac{(a-m_A)^2}{2\sigma_A^2}}\mathrm{d}a$$

$$
\begin{aligned}
&= e^{\frac{\sigma_A^2}{2}-m_A} Q\left(\sigma_A - \left(\frac{m_A - e^{A_{\text{out}}}}{\sigma_A}\right)\right) \\
&= 10^{-m_B/10} e^{\left(\frac{\ln 10}{10}\right)^2 \frac{\sigma_B^2}{2}} Q\left(\frac{\ln 10}{10}\sigma_B - Q_{\text{inv}}(P_{\text{out}})\right)
\end{aligned} \tag{52}
$$

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