Iterative Sparse Channel Estimation for Acoustic OFDM Systems

Sayedamirhossein Tadayon and Milica Stojanovic
Northeastern University, Boston, MA, USA

***Abstract*—We propose a method for channel estimation in orthogonal frequency division multiplexing (OFDM) systems with an array of receiving elements. In contrast to traditional methods, which target the equivalent sample-spaced channel taps, this method targets the physical propagation paths whose delays are not restricted to the sample-spaced grid. The path delays are estimated jointly with the path gains and the angles of arrival, exploiting coherence between the array elements. Numerical results illustrate superior performance as compared to tap-based channel estimation, as well as additional gain available from spatial correlation across the array elements.**

I. INTRODUCTION

Channel estimation for multi-carrier (OFDM) acoustic systems has been studied extensively in recent years e.g. [1],[2]. These studies have recognized the fact that acoustic channels are sparse, and have put forth a number of channel estimation algorithms that take advantage of this fact.

The majority of the existing algorithms target estimation of sample-spaced channel taps, with sampling at the basic rate equal to the system bandwidth. Over-sampling the channel results in a greater total number of model taps, but improves resolution, which may in turn boost the sparseness. Taking advantage of this fact, [1] offered two algorithms for channel estimation, one based on the greedy orthogonal matching pursuit (OMP) and the other based on the basis pursuit (BP). These methods showed a marked improvement in application to both synthetic and real data; however, the associated computational complexity, which comes from using finer over-complete dictionaries, in not negligible. Additionally, a strong effect of diminishing returns is observed as the granularity (oversampling ratio) is increased.

To address these issues, we formulate the problem of sparse channel estimation in a different manner. We target a continuum of path delays, eliminating the sample-spaced model and focusing instead on processing a transformed version of the signal observed over all the carriers spanning the system bandwidth. We draw on the ideas of [3], where we proposed the basic approach of path identification for a single-element receiver. Unlike the sparse identification methods with over-complete dictionaries, the resolution and coverage in delay that this method provides can be increased arbitrarily without a penalty on performance and without a prohibitive cost to complexity.

In this paper, we extend the path identification method to the framework of a receiver array. In doing so, we target an additional degree of physical sparsity that is present in the spatial dimension, where each propagation path is associated with an angle of arrival. The result is a two-dimensional (delay-angle) path identification (PI) algorithm that we present here.

In Sec. II, we introduce the system and channel models. Sec. III briefly explores the discrete-delay channel model and the associated estimation algorithms. Sec. IV outlines the path identification method, while Sec. V details its multi-channel implementation. Sec. VI presents the results of numerical simulations which quantify the performance with a varying number of carriers and receiving elements. We conclude in Sec. VII.

II. SYSTEM AND CHANNEL MODEL

We consider an OFDM system with K carriers within a total bandwidth B. We denote by f_0 and $\Delta f = B/K = 1/T$ the frequency of the first carrier and the carrier separation, respectively. We assume the use of a cyclic prefix which preserves the carrier orthogonality and eliminates inter-block interference. The channel is modeled as slowly fading and constant during one OFDM block. Under these assumptions, the signal received on the k-th carrier can be described as

$$y_k = d_k H_k + z_k,\ k = 0,\ldots,K-1 \tag{1}$$

where d_k is the transmitted data symbol (we will assume a unit-variance PSK alphabet), H_k is the channel frequency response at the frequency of the k-th carrier, and z_k is the circularly symmetric, zero-mean additive Gaussian noise of variance σ_z^2.

We model the propagation channel as

$$H(f) = \sum_p h_p e^{-j2\pi f \tau_p} \tag{2}$$

where h_p and τ_p represent the path gains and delays, respectively. At the carrier frequencies $f_k = f_0 + k\Delta f$, we then have

$$H_k = \sum_p \underbrace{h_p e^{-j2\pi f_0 \tau_p}}_{c_p} e^{-j2\pi k \Delta f \tau_p},\ k = 0,\ldots,K-1. \tag{3}$$

III. CHANNEL ESTIMATION BASED ON THE DISCRETE-DELAY MODEL

Conventionally, channel estimation is based on the discrete-delay model, which relates the channel coefficients H_k to the sample-spaced taps b_l as

$$H_k = \sum_l b_l e^{-j2\pi k \Delta f l T_s},\ k = 0,\ldots,K-1 \tag{4}$$

When $T_s = T/K$, we have the usual DFT relationship between the vectors of coefficients $\mathbf{H}$ and $\mathbf{b}$,

$$\mathbf{H} = \mathbf{F}_K \mathbf{b} \tag{5}$$

where $\mathbf{F}_K$ is the $K \times K$ Fourier matrix. If the multipath spread T_{mp} is within LT/K, then we can also write

$$\mathbf{H} = \mathbf{F}_{KL} \mathbf{b}_L \tag{6}$$

where $\mathbf{F}_{KL}$ contains only the first L columns of the Fourier matrix $\mathbf{F}_K$, and $\mathbf{b}_L$ contains only the first L elements of $\mathbf{b}$.

Assuming without the loss of generality that all K data symbols are available for channel estimation (e.g. correct symbol decisions, or all-pilots in an initial block), the input to the channel estimator is given by

$$\mathbf{x} = \mathbf{H} + \mathbf{z} \tag{7}$$

where $\mathbf{z}$ represents an equivalent noise vector. The conventional LS estimate is then given by

$$\hat{\mathbf{b}}_L = \frac{1}{K} \mathbf{F}_{KL}^H \mathbf{x}, \quad \hat{\mathbf{H}} = \mathbf{F}_{KL} \hat{\mathbf{b}}_L \tag{8}$$

In many practical applications, the multipath channel can be considered as sparse, and the greedy OMP or the BP algorithms can be used to estimate the channel response [**?**], [1]. The OMP algorithm identifies the dominant channel taps sequentially, selecting in each iteration one column of the over-complete dictionary $\mathbf{F}_{KL}$ that correlates best with the residual from the previous iteration, and recomputes the coefficients by solving a constrained least-squares problem to fit the observations [4]. In contrast, BP uses mixed-norm minimization to estimate the channel taps as

$$\hat{\mathbf{b}}_L = \arg\min_{\mathbf{b}_L} \|\mathbf{x} - \mathbf{F}_{KL} \mathbf{b}_L\|_2^2 + \lambda \|\mathbf{b}_L\|_1 \tag{9}$$

where the parameter λ is tuned according to the variance of the noise.

These algorithms can also be applied in "super-resolution" form, using sub-sample spacing, i.e. $T_s = T/KI$, where I is an integer (e.g. 2,4, 8).

IV. CHANNEL ESTIMATION BASED ON THE PHYSICAL MODEL

Referring to (3) as the physical channel model, we re-write it in matrix form as

$$\mathbf{H} = \sum_p c_p \mathbf{s}_K(2\pi \Delta f \tau_p) \tag{10}$$

where $\mathbf{s}_K(2\pi\Delta f \tau) = \begin{bmatrix} 1 & e^{-j2\pi\Delta f \tau} & \dots & e^{-j2\pi(K-1)\Delta f \tau} \end{bmatrix}^T$ represents the steering vector at an arbitrary delay τ.

Consider now the following operation performed on the noisy channel observation $\mathbf{x}$:

$$r(\tau) = \frac{1}{K} \mathbf{s}_K^H(2\pi\Delta f \tau) \mathbf{x} \tag{11}$$

This operation can be thought of as steering in the frequency domain (across carriers). The newly formed signal is

$$r(\tau) = \sum_p c_p g_K \left(2\pi\Delta f \left(\tau - \tau_p\right)\right) \tag{12}$$

where $g_K(\phi) = \frac{1}{K} \sum_{k=0}^{K-1} e^{jk\phi}$ is a *known* signature waveform.

The signal $r(\tau)$ serves as the input to the path identification algorithm. The algorithm operates recursively, identifying in each iteration the delay of the next-strongest path in a manner similar to OMP [3].

V. MUTICHANNEL PROCESSING

When multiple receiving elements are available, two situations are possible: one in which the array elements see uncorrelated channel responses, and another in which the channels responses are correlated. In the first case, channel estimation must be accomplished element-by-element. This is done simply by applying the path identification algorithm to each element individually. In the second case, correlation between the elements can be exploited. Below, we elaborate on the second case.

Specifically, we assume that the following model holds for the channel frequency response observed at the m-th receiving element and the k-th carrier (far field, plane wave propagation):

$$H_k^m = \sum_p h_p e^{-j2\pi k \Delta f \tau_p^m} \tag{13}$$

where $\tau_p^m = \tau_p + m\Delta\tau_p$ for $m = 0, \dots, M-1$.

The correlation between the receiving elements is embodied in the differential delays $\Delta\tau_p$ which are related to the paths' angles of arrival θ_p. If the arrival angles are measured with respect to the horizontal (e.g. an arrival angle of 0 corresponds to the direct path between the transmitter and receiver placed at the same depth), for the array elements separated by d and the propagation speed c, we have that

$$\Delta\tau_p = \frac{d}{c} \sin\theta_p \tag{14}$$

The channel coefficients can be expressed as[1]

$$H_k^m = \sum_p \underbrace{h_p e^{-j2\pi f_0 \tau_p^m}}_{c_p^m = c_p e^{-j2\pi f_0 m \Delta\tau_p}} e^{-j2\pi k \Delta f \tau_p^m} \tag{15}$$

Collecting all the carriers into a vector, we form

$$\mathbf{x}^m = \sum_p c_p^m \mathbf{s}_K(2\pi\Delta f \tau_p^m) + \mathbf{z}^m \tag{16}$$

where $\mathbf{z}^m$ is the noise pertaining to the m-th receiver.

Steering across carriers now yields a new signal for each array element,

$$\begin{aligned} r^m(\tau) &= \frac{1}{K} \mathbf{s}_K^H(2\pi\Delta f \tau) \mathbf{x}^m \\ &= \sum_p c_p^m g_K \left(2\pi\Delta f \left(\tau - \tau_p^m\right)\right) + z^m(\tau) \end{aligned} \tag{17}$$

If the path identification algorithm is applied to each receiving element individually, it will yield the estimates of c_p^m and τ_p^m. These estimates are obtained without taking advantage of the inter-element correlation. Hence, it is conceivable that an estimator that takes this correlation into account might perform better. To arrive at such an estimation algorithm, we form a

[1]Here, ^{m} stands as a superscript.

vector $\mathbf{r}(\tau,\Delta\tau)$ of array signal with the m-th element given by $r^m(\tau+m\Delta\tau)$.

A new steering operation is now applied across the array elements to yield a signal

$$r(\tau,\Delta\tau) = \frac{1}{M}\mathbf{s}_M^H(2\pi f_0\Delta\tau)\mathbf{r}(\tau,\Delta\tau)$$
$$=\sum_p c_p g_{K,M}(2\pi\Delta f(\tau-\tau_p), 2\pi f_0(\Delta\tau-\Delta\tau_p)) + z(\tau,\Delta\tau) \quad (18)$$

where

$$g_{K,M}(\phi,\psi) = \frac{1}{M}\sum_{m=0}^{M-1} g_K(\phi + m\underbrace{\frac{\Delta f}{f_0}}_{\varepsilon}\psi)e^{jm\psi} \quad (19)$$

is the composite signature waveform. This waveform is a function of two arguments, one related to the path delays, and another related to the path angles. Fig. 1 illustrates the composite signature waveform. Clearly, this function exhibits a peak at the origin, and we use this fact to identify the delay-angle pairs corresponding to the propagation paths.

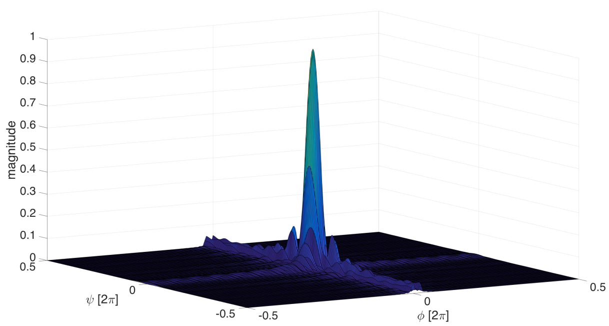

Fig. 1. Composite signature waveform (one period, magnitude). $K=128$, $M=12$, $\varepsilon=0.0037$.

The procedure for identifying the path delays and angles is similar to the previous procedure where only the path delays had to be identified. The new procedure differs in the fact that a one-dimensional search for the delays is now replaced by a two-dimensional search for the delays and angles.

The search is again based on the fact that the signature waveform is a known function. It begins by identifying the maximum of $|r(\tau,\Delta\tau)|$ which points to the strongest path coefficient, and proceeds to remove it from the signal. It then continues recursively, removing the next strongest path in every iteration. Table I contains the formal steps of the 2D path identification algorithm.

Setting $\Delta\tau=0$, i.e. omitting the operations in the angle domain, yields the 1D path identification algorithm. With delay resolution $T_s = T/KI$ and multipath window $L = \lceil T_g/T_s \rceil$, path identification requires $O(KLI)$ operations to obtain the input signal $r(\tau)$ and $O(LI)$ computations per iteration. In contrast, OMP requires $O(KLI + Kp + Kp^2 + Kp^3)$ operations in iteration p, while the complexity of BP is $O(K^2(LI)^{3/2})$ [5].

TABLE I
2D PATH IDENTIFICATION ALGORITHM

INPUT:
- signal $r(\tau,\Delta\tau)$ as expressed in (18)
- number of channel paths P (or threshold η)

OUTPUT:
- estimated path delays $\hat{\tau}_p$
- estimated differential delays $\hat{\Delta\tau}_p$
- estimated path gains $\hat{c}_p$

INITIALIZATION:
$r_0(\tau,\Delta\tau) = r(\tau,\Delta\tau)$

PROCEDURE:
while $p \le P$ (or $|r_p(\tau,\Delta\tau)| > \eta$) **do**
$(2\pi\Delta f\hat{\tau}_p, 2\pi f_0\Delta\hat{\tau}_p) = \arg\max_{\tau,\Delta\tau}|r_p(\tau,\Delta\tau)|$
$\hat{c}_p = r(\hat{\tau}_p, \hat{\Delta\tau}_p)$
$r_{p+1}(\tau,\Delta\tau) = r_p(\tau,\Delta\tau) - \hat{c}_p g_{K,M}(2\pi\Delta f(\tau-\hat{\tau}_p), 2\pi f_0(\Delta\tau-\Delta\hat{\tau}_p))$
end while

VI. SIMULATION RESULTS

To assess the system performance, we focus on an example of an OFDM system with parameters specified in Table II. We use a three-path channel with delays 0, 4.2 msec and 9.5 msec, and angles of arrival $0°$, $9°$ and $-13°$, respectively. The path gains are modeled as complex-valued Gaussian random variables with zero mean and variance $\sigma_p^2 = Ce^{-\tau_p}$, where the constant C is determined such that $\sum_p \sigma_p^2 = 1$. This model is chosen for illustration purposes, and represents a simplified case of the experimentally verified non-zero-mean model [6].

TABLE II
PARAMETERS OF NUMERICAL SIMULATION

lowest frequency (f_0)	10.5 kHz
bandwidth (B)	5 kHz
number of carriers (K)	$\{128, 256\}$
number of receiving elements (M)	$\{4, 8, 12\}$
modulation type	QPSK
propagation speed (c)	1500 m/sec
inter-element distance (d)	6 cm

We assess the algorithm performance by measuring the mean squared error (MSE) on one of the receiving elements,

$$MSE^m = \frac{1}{K}\mathbb{E}\{\|\mathbf{H}^m - \hat{\mathbf{H}}^m\|^2\} \quad (20)$$

Fig. 2 shows the normalized MSE versus the signal-to-noise ratio SNR$=\frac{1}{\sigma_z^2}$. The results are shown for the 1D and the 2D path identification algorithms, as well as for LS, OMP and BP as comparison benchmarks. Clearly, 2D path identification outperforms all the other methods. Its performance owes to the fact that *each* channel is estimated using *all* the array observations, i.e. that spatial coherence is exploited. Standard, tap-based channel estimation algorithms could also be cast into the array framework, but their computational complexity would grow.

In Fig. 3, we investigate the effect of the number of receiving elements on the performance of 2D path identification. We note that by increasing the number of receiving elements from

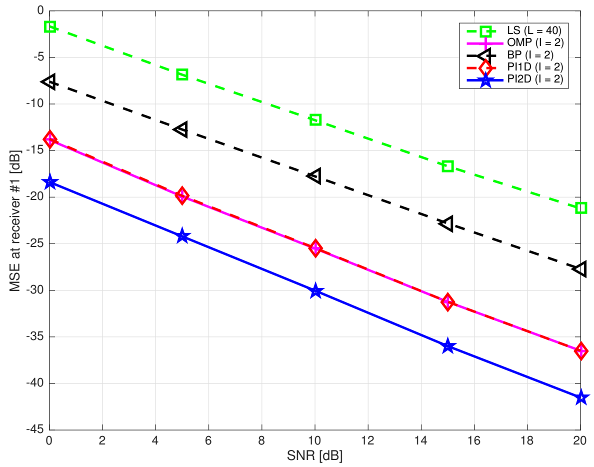

Fig. 2. Mean-squared error for different estimators. 2D path identification operates with $M = 4$ receiving elements. Resolution is 0.1 msec in the delay domain and 1.146° in the angle domain. The number of carriers is $K = 128$. The results represent an average of 1000 simulation runs.

one to 4, 8 and 12, yields an average effective SNR gain of 4.6, 8.6 and 11.2 dB, respectively.

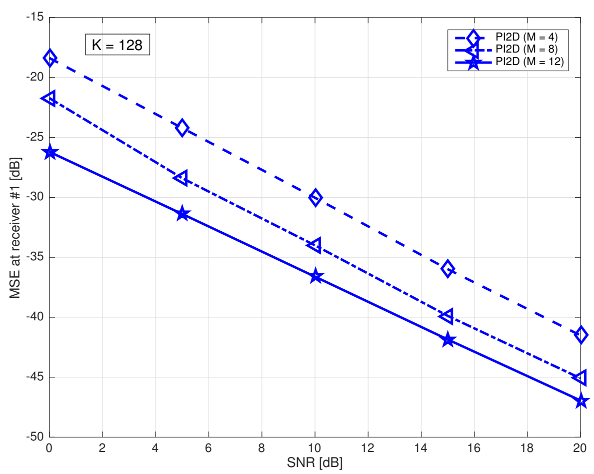

Fig. 3. Effect of the array size M on the performance of 2D path identification.

Finally, Fig. 4 illustrates the system performance for a varying number of carriers K. As the number of carriers grows, steering in the frequency domain improves, benefiting both 1D and 2D path identification. Note, however, that a practical system will have a limit on the number of carriers that can be used within a given bandwidth before the the inter-carrier interference starts to degrade the performance.

VII. Conclusion

We addressed the issue of channel estimation in OFDM systems, targeting explicitly the physical propagation paths, rather than the taps of an equivalent discrete-delay impulse response model. The basic difference between the two approaches is that the former allows the path delays to have a continuum of values, while the latter restricts the taps to a pre-defined quantization grid. As a result, the number of taps that have to be estimated is typically greater than the number of paths.

By employing steering operations across carriers and receiving elements, the channel estimation problem was cast into a framework where path delays, angles and gains can be identified in an iterative manner. Numerical results indicate superiority of the proposed method over the standard tap-based sparse channel estimation techniques.

Future research will focus on adaptive algorithm operation for time-varying channels, applications to real data, and performance assessment through bit error rate analysis.

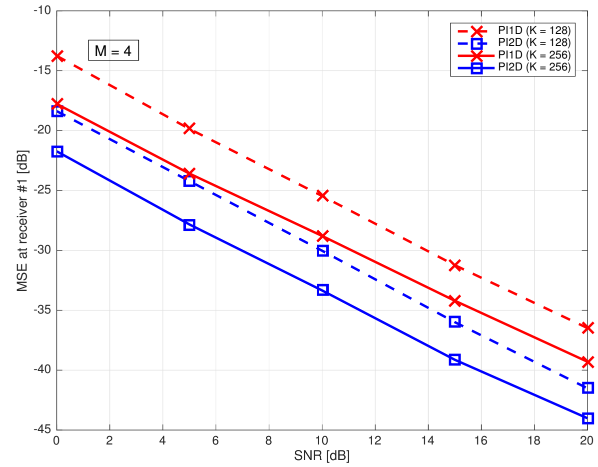

Fig. 4. Effect of the number of carriers K on the performance of path identification.

Acknowledgement

This work was supported by the following grants: NSF CNS-1212999, ONR N00014-15-1-2550.

References

[1] C. R. Berger, S. Zhou, J. C. Preisig, and P. Willett, "Sparse channel estimation for multicarrier underwater acoustic communication: From subspace methods to compressed sensing," *Signal Processing, IEEE Transactions on*, vol. 58, no. 3, pp. 1708–1721, 2010.

[2] A. Radosevic, R. Ahmed, T. M. Duman, J. G. Proakis, and M. Stojanovic, "Adaptive OFDM modulation for underwater acoustic communications: Design considerations and experimental results," *Oceanic Engineering, IEEE Journal of*, vol. 39, no. 2, pp. 357–370, 2014.

[3] M. Stojanovic and S. Tadayon, "Estimation and tracking of time-varying channels in ofdm systems," in *Communication, Control, and Computing (Allerton), 2014 52nd Annual Allerton Conference on*, pp. 116–122, IEEE, 2014.

[4] W. Li and J. C. Preisig, "Estimation of rapidly time-varying sparse channels," *Oceanic Engineering, IEEE Journal of*, vol. 32, no. 4, pp. 927–939, 2007.

[5] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *Information Theory, IEEE Transactions on*, vol. 53, no. 12, pp. 4655–4666, 2007.

[6] P. Qarabaqi and M. Stojanovic, "Statistical characterization and computationally efficient modeling of a class of underwater acoustic communication channels," *Oceanic Engineering, IEEE Journal of*, vol. 38, no. 4, pp. 701–717, 2013.