Frequency Offset Compensation for Acoustic OFDM Systems

Amir Tadayon *Student Member, IEEE* and Milica Stojanovic *Fellow, IEEE*
Northeastern University, Boston, MA, USA

***Abstract*—This paper addresses the problem of compensating for motion-induced Doppler frequency offset in multicarrier acoustic communication systems based on orthogonal frequency division multiplexing (OFDM). In mobile acoustic systems, Doppler effect can be sever enough that the received OFDM signal experiences non-negligible frequency offsets even after initial resampling. To target these offsets, a practical method based on a hypothesis-testing approach is proposed. The method relies on differentially coherent detection which keeps the receiver complexity at a minimum and requires only a small pilot overhead. Differential encoding is applied across carriers, promoting the use of a large number of closely spaced carriers within a given bandwidth. This approach simultaneously supports frequency-domain coherence and efficient use of bandwidth for achieving high bit rates. While frequency synchronization capitalizes on differentially coherent detection framework, it can also be used as a pre-processing stage in coherent receivers without creating undue complexity. Using the experimental data transmitted over a 3-7 km shallow water channel in the 10.5-15.5 kHz acoustic band, we study the system performance in terms of data detection mean squared error (MSE) and bit error rate (BER), and show that the proposed method provides excellent performance at low computational cost.**

I. Introduction

Multicarrier modulation in the form of orthogonal frequency division multiplexing (OFDM) is an attractive method for data transmission over frequency-selective channels due to its abality to achieve high bit rates at reasonably low computational loads [1]–[3]. This fact motivates the use of OFDM in mobile acoustic communications where the channel exhibits long multipath delays but each narrowband carrier only experiences flat fading, thus eliminating the need for time domain equalizers.

The major problem in applying OFDM to acoustic channels is the Doppler distortion due to relative motion between the transmitter and receiver, which causes non-uniform frequency shifting across the acoustic signal bandwidth. For the relative transmitter/receiver velocity v and the propagation speed c (nominally 1500 m/s), Doppler scaling occurs at the rate $a = v/c$. In highly mobile scenarios, Doppler frequency scaling is effectively seen as a time-varying channel distortion which adversely affects the performance of OFDM systems as it causes loss of orthogonality between the carriers. To mitigate the resulting distortion, front-end resampling must be performed [1]–[3]. Coarse resampling is typically performed on an entire frame of OFDM blocks, and may leave individual blocks within a frame exposed to different frequency offsets. These offsets, if left uncompensated, can have a detrimental impact on data detection.

In this paper, we target these frequency offsets through a hypothesis testing approach. The approach is based on differentially coherent detection which keeps the receiver complexity at a minimum and requires only a very low pilot overhead. Differential encoding is applied across carriers, promoting the use of a large number of carriers within a given bandwidth [1]. This approach simultaneously supports frequency-domain coherence and efficient use of bandwidth for achieving high bit rates. The technique is demonstrated on experimental data from the Mobile Acoustic Communication Experiment (MACE 2010) showing excellent results in situations with Doppler frequency offsets on the order of a carrier spacing. In the MACE10 experiment, the transmitter moves at a relative speed of 0.5-1.5 m/s with respect to the receiver, and the OFDM blocks containing up to 2048 QPSK modulated carriers occupy the acoustic frequency range between 10.5 and 15.5 kHz.

The rest of the paper is organized as follows. In Sec. II, we introduce the signal and system model. Sec. III details the proposed method for compensating frequency offsets. Sec. IV contains the results of experimental data processing. Sec.V contains the conclusions.

II. Signal and System Model

We consider an OFDM system with M_r equi-spaced receivers and K carriers within a total bandwidth B. Let f_0 and $\Delta f = B/K$ denote the first carrier frequency and carrier spacing, respectively. We assume the use of zero-padding in the transmitter along with the overlap-and-add procedure at the receiver [4]. The transmitted OFDM block is then given by

$$s(t) = Re\left\{\sum_{k=0}^{K-1} d_k e^{2\pi i f_k t} g(t)\right\}, \quad t \in [0,\ T+T_g] \quad (1)$$

where $T = 1/\Delta f$ is the OFDM symbol duration, T_g is the zero guard interval which is assumed to be at least as long as the multipath spread of the channel, $T_g \geq T_{mp}$, and $g(t)$ describes the zero-padding operation, i.e. $g(t) = 1$ for $t \in [0, T]$ and 0 otherwise. The data symbols d_k, which modulates the kth carrier of frequency $f_k = f_0 + k\Delta f$, belongs to a unit amplitude phase shift keying alphabet (PSK).

The OFDM system transmits N_d data symbols in an OFDM frame that includes a preamble, $N_b = N_d/K$ OFDM blocks, and a postamble. The synchronization preamble and postamble are short signals formed from a pseudo-noise sequence mapped to a unit-amplitude binary PSK alphabet.

The transmitted signal passes through a multipath acoustic channel whose impulse response can be modeled as

$$h(\tau, t) = \sum_p h_p(t)\delta(\tau - \tau_p(t)) \tag{2}$$

where $h_p(t)$ and $\tau_p(t)$ represent the gain and delay of the pth path, respectively. We isolate a common Doppler scaling factor a such that $\tau_p(t) \approx \tau_p - at$, and further assume that the path gains are slowly varying such that $h_p(t) \approx h_p$ for the duration of one OFDM block. With these notions, we can rewrite (2) as

$$h(\tau, t) \approx \sum_p h_p \delta(\tau - \tau_p + at) \tag{3}$$

Assuming a multi-element receiver with M_r elements, the signal received on the mth element is given by

$$\tilde{r}_m(t) = \sum_p h_p^{(m)} s((1+a)t - \tau_p^{(m)}) + n_m(t) \tag{4}$$

where the Doppler scaling factor a is assumed to be the same for all the receiving elements, and $n_m(t)$ is the additive noise with power spectral density (PSD) $N_0/2$. The noise on different receiving elements is assumed to be uncorrelated, i.e. $\mathbb{E}\{n_i(t)n_j(t)\} = \frac{N_0}{2}\delta_{i,j},\ \forall i, j = 1, \ldots, M_r$.

Frame synchronization is performed using the method proposed in [1]. Front-end resampling is then applied to compensate for the time compression/dilation that the received signal experiences. To obtain a rough estimate of Doppler scaling factor, we measure the length of the received frame as $T_{rx} = T_{tx} + \Delta\tau$ where T_{tx} is the transmitted frame duration and $\Delta\tau$ represents the amount of time compression/dilation. Time compression/dilation is obtained as $\Delta\tau = \arg\max_\tau |R_{pre,post}(\tau)|$ where $R_{pre,post}(\tau)$ is the correlation between the received preamble and postamble signals (its magnitude peaks at lag zero if there is no time compression/dilation). Hence, it follows that $\hat{a} = T_{tx}/T_{rx} - 1$, and the resampled received signal is obtained as $r_m(t) = \tilde{r}_m(t/(1 + \hat{a}))$.

After frame synchronization, initial resampling and downshifting by the lowest carrier frequency f_0, the received signal on the mth receiving element is modeled as

$$v_m(t) = e^{i\beta t} \sum_{k=0}^{K-1} H_k^m d_k e^{2\pi i k \Delta f t} + w_m(t), \quad t \in [0, T] \tag{5}$$

where β is the unknown frequency offset assumed common for all M_r receiving elements, H_k^m is the channel frequency response at the kth carrier of the mth receiving elements and $w_m(t)$ is the additive complex Gaussian noise with PSD N_0 per complex dimension. To obtain (5), we assumed that the ratio $(1+a)/(1+\hat{a})$ is close to 1 and we invoked the fact that $T \gg T_{mp}$ for a properly designed OFDM system. Assuming the same gross frequency offset β for all receiving elements is plausible when the elements are co-located, and it helps to promote the multichannel processing gain.

The model (5) captures rough frequency shifting and serves as a starting point in developing the method for frequency offset compensation. The finer points of frequency shift changing across the bandwidth are left to post-FFT processing.

III. Frequency Offset Compensation

We focus on frequency offset compensation which employs differentially coherent detection. In this approach, several hypothesized values of the frequency offset are used, e.g. in steps of $\Delta f/10$, and differential maximal ratio combining is performed for each hypothesized value. Specifically, let us assume that the M_r signals are compensated by some hypothesized value $\hat{\beta}$, and that demodulation is performed on all the receiving elements to yield

$$y_k^m = \int_T v_m(t) e^{-i\hat{\beta}t} e^{-2\pi i k \Delta f t} dt \tag{6}$$

where $k = 0, \ldots, K-1$ and $m = 1, \ldots, M_r$. Arranging the signals corresponding to carrier k into a vector $\mathbf{y}_k$, the estimates of the differentially-encoded data symbols $b_k = d_{k-1}^* d_k$ are obtained as

$$\hat{b}_k = \frac{\mathbf{y}_{k-1}^H \mathbf{y}_k}{\mathbf{y}_{k-1}^H \mathbf{y}_{k-1}}, \quad k = 0, \ldots, K-1 \tag{7}$$

where we implicitly assume that the channel frequency response changes slowly from one carrier to the next, i.e. $H_{k-1}^m \approx H_k^m$, $\forall k = 1, \ldots, K-1$ and $m = 1, \ldots, M_r$.

Using equally-spaced pilot data symbols $b_k, k \in \mathcal{K}_p$, the composite error is formed as

$$E(\hat{\beta}) = \sum_{k \in \mathcal{K}_p} |b_k - \hat{b}_k|^2 \tag{8}$$

and the estimate $\hat{\beta}^\star$ is obtained as

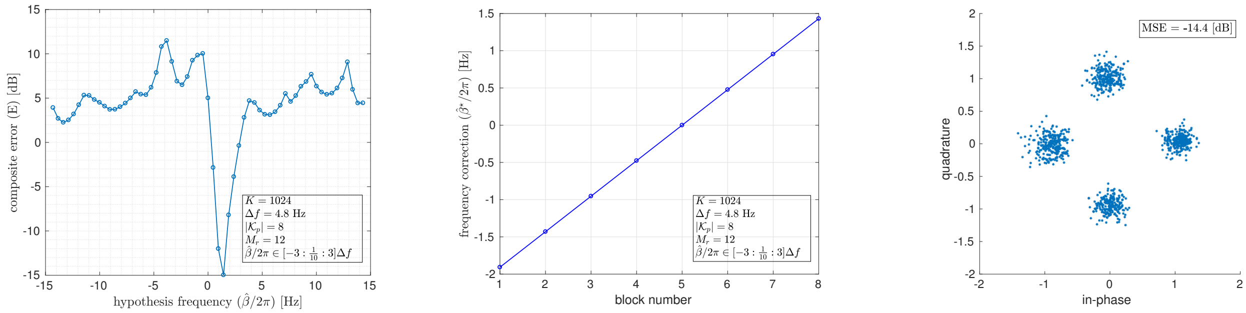
$$\hat{\beta}^\star = \arg\min_{\hat{\beta}} E(\hat{\beta}) \tag{9}$$

In Fig. 1, we illustrate how the method works by applying it to an OFDM frame with 8 blocks and $K = 1024$ carriers from the experimental recordings. Shown in the figure are the last block's composite error as a function of the hypothesized frequency offset values $\hat{\beta}/2\pi$, the estimated frequency offset $\hat{\beta}^\star/2\pi$ obtained for the 8 blocks in the underlying frame, and the scatter plot of the detected data symbols in the last block. In the next section, we study the performance of the system using the entire set of signals transmitted over several hours.

The running time of the algorithm is dominated by the number of FFTs which is equal to the number of hypothesized values $N_{\hat{\beta}}$. The total cost of the algorithm is $O(N_{\hat{\beta}} K \log(K))$. This cost can further be reduced by exploiting the channel coherence. Namely, capitalizing on the fact that the channel is not changing much from one block to the next, we can pre-compensate the signal in the next block by the frequency offset estimate obtained in the current block, then use a shorter hypothesis interval. Such an approach will effectively reduce the overall computational complexity.

IV. Experimental Result

To asses the system performance, we focus on the experimental data from the Mobile Acoustic Communication Experiment (MACE'10) which took place off the coast of Martha's Vineyard, Massachusetts, in June 2010. The experimental signals, whose parameters are given in Table I, were transmitted using the acoustic frequency range between



(a) Last block's composite error (b) Frequency correction (c) Last block's scatter plot

Fig. 1. Performance illustration for an OFDM frame with $K = 1024$ carriers and 8 blocks. Shown are the composite error obtained for the last block *(a)*, frequency correction *(b)*, and the last block's scatter plot *(c)*. Only 8 pilots are used and the hypothesized values range from $-3\Delta f$ to $3\Delta f$ in steps of $\Delta f/10$. There are no symbol decision errors in the scatter plot shown.

TABLE I
MACE'10 SIGNAL PARAMETERS. THE GUARD INTERVAL IS $T_g = 16$ MSEC. THE TOTAL BANDWIDTH IS $B = 5$ KHZ AND THE LOWEST CARRIER FREQUENCY IS $f_0 = 10.5$ KHZ. THE BANDWIDTH EFFICIENCY IS CALCULATED ASSUMING 8 PILOTS.

number of carriers K	64	128	256	512	1024	2048
number of blocks per frame N_b	128	64	32	16	8	4
carrier spacing Δf [Hz]	78.1	39.1	19.5	9.8	4.9	2.4
bit rate [kbps]	4.4	6.2	7.6	8.6	9.3	9.6
bandwidth efficiency [bps/Hz]	0.76	1.14	1.4	1.7	1.84	1.92

10.5 kHz and 15.5 kHz. The receiver array of 12 equally-spaced elements spanning a total linear aperture of 1.32 m was suspended at the depth of 40 m, and the transmitter was towed at the depth of 40-60 m. The water depth was approximately 100 m, and the transmission distance varied between 3 km and 7 km. More details about the experiment can be found in [1].

The experiment consisted of multiple repeated transmissions, each containing all the OFDM signals listed in Table I. There was a total of 52 transmissions spanning 3.5 hours of recording. During this time, the transmitting station moved away and towards the receiving station, at varying speeds ranging from 0.5 m/s to 1.5 m/s. The results provided in this section are obtained from all 52 transmissions.

We demonstrate the performance of the proposed scheme for frequency offset compensation in terms of data detection mean-squared error (MSE) and average execution time $\bar{T}_{exe}$ which is deemed a practical indicator of the algorithm complexity. We also report on the estimated cumulative density function (CDF) of the MSE measured in each signal frame. Furthermore, we show the bit error rate (BER) and block error rate (BLER) of the system using low-density parity check (LDPC) codes with various code rates.

The MSE is measured in the n-th block of the i-th frame as

$$MSE^i(n,K) = \frac{1}{K-1}\sum_{k=1}^{K-1} |b_k^i(n) - \hat{b}_k^i(n)|^2 \tag{10}$$

and the MSE per frame is obtained as

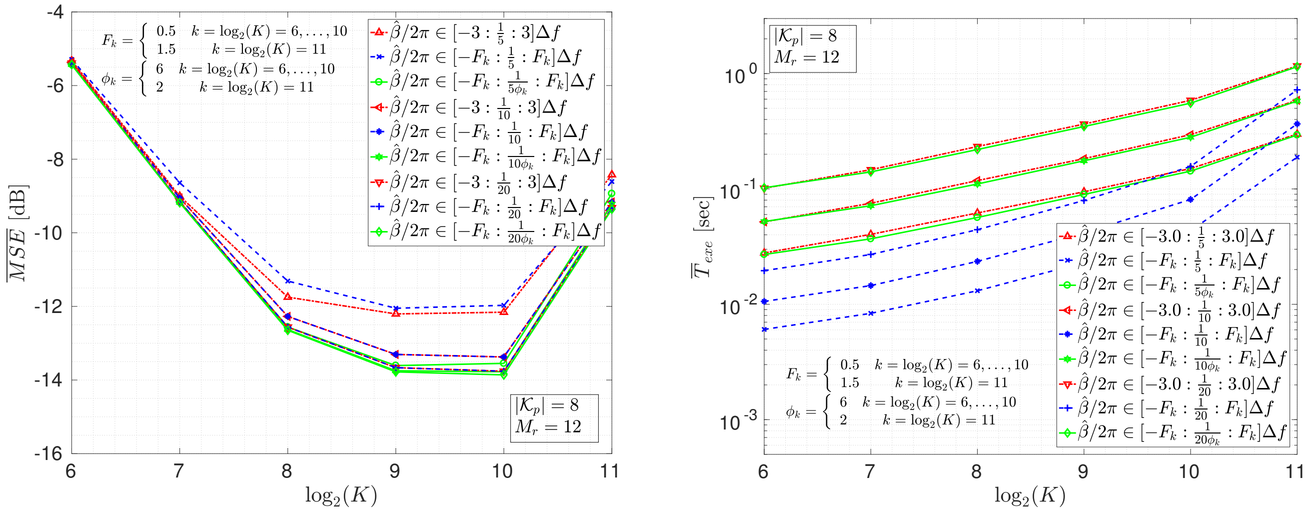
$$MSE^i(K) = \frac{1}{N_b}\sum_{n=1}^{N_b} MSE^i(n,K) \tag{11}$$

The average over all 52 frames is

$$\overline{MSE}(K) = \frac{1}{52}\sum_{i=1}^{52} MSE^i(K) \tag{12}$$

Note that due to the random channel variation and a finite number of measurements, each of these quantities is a random variable.

Fig. 2 illustrates the average MSE (Fig. 2a) and the average execution time (Fig. 2b) of the algorithm as a function of the number of carriers K (log scale) for three situations, each of which includes three hypothesis intervals: S1) fixed hypothesis intervals $[-3\Delta f, 3\Delta f]$ with resolution factors 5, 10 and 20 for all the blocks in a frame, S2) hypothesis intervals from the second block and on are changed to shorter intervals $[-0.5\Delta f, 0.5\Delta f]$ and $[-1.5\Delta f, 1.5\Delta f]$ for $\log_2 K = 6, \ldots, 10$ and $\log_2 K = 11$, respectively, while the resolution factors are the same as those in the first set, and S3) the same hypothesis intervals as in S2 but the resolution is increased by a factor of 6 for $\log_2 K = 6, \ldots, 10$ and by a factor of 2 for $\log_2 K = 11$. Each point in these plots is obtained by averaging over all carriers, blocks, and 52 frames transmitted. Fig 2a in conjunction with Fig. 2b demonstrate the trade-off between performance and complexity. Fig 2a clearly shows that increasing the resolution factor improves the MSE performance. Taking the hypothesis intervals in S1 as an example, we observe that by increasing the resolution factor from 5 to 10 for the case where $\log_2(K) = 10$, we obtain 1.2 dB gain in the MSE performance. However, further increase in resolution factor from 10 to 20 does not reveal any gain in the MSE performance while it doubles the computational complexity as demonstrated in Fig. 2b. From Fig. 2a, it is clear that using the hypothesis intervals in S2



(a) MSE

(b) Execution time

Fig. 2. *(a)* Average MSE versus the number of carriers K. *(b)* Average execution time as a function of the number of carriers (log scale). The dash-dotted red curves, dashed blue curves and solid green curves correspond to the hypothesis intervals in S1, S2 and S3, respectively. Only 8 pilots are used to form the composite error E. All 12 receivers are used to perform DMRC (7).

we achieve the same MSE performance as we do using the intervals in S1; however, using the intervals in S2 we reduce the computational cost of the algorithm by a factor of 4 as shown in Fig. 2b. The hypothesis intervals in S3, which have the same range as those in S2 but with finer resolutions, result in similar MSE performance as those in S2, but at the same cost as those in S1. In a nutshell, using the intervals in S2, we obtain the best trade-off between performance and complexity. Thus, the rest of our analysis considers only the intervals in S2.

Fig. 2a also shows that increasing the number of carriers from 64 to 1024 (from $\log_2(K) = 6$ to 10) significantly improves the MSE perfromance (as well as bandwidth efficiency) of the system since a larger number of carriers leads to a higher frequency-domain coherence which is essential for successful differentially-coherent data detection [1]. The apparent deterioration in performance for the OFDM frames with $\log_2(K) = 11$ can be explained by the increased block duration that nudges the temporal coherence of the channel.

Fig. 3 illustrates the estimated cumulative density function of the MSE per block. This result refers to $K = 1024$ carries and includes the 52 frames transmitted over 3.5 hours. Using the hypothesis intervals with range $[-0.5\Delta f, 0.5\Delta f]$ and resolution factors 5, 10 and 20, the system delivers MSE below 12 dB for 61%, 84% and 87% of the OFDM blocks, respectively. Fig. 4 illustrates the MSE performance as a function of the number of receiving elements M_r which are chosen among the 12 available elements. The receiving elements are maximally equally-spaced.

In Fig. 5, we demonstrate the performance of the system in terms of average bit error rate (BER) using regular low-density parity check (LDPC) codes with various code rates range from 0.1 to 1. The codewrod length is $N = 2K$, thus, each codeword constitutes an OFDM block. The column weight of the $M \times N$ parity check matrix is $w_c = 3$ for all the code rates considered, and the row weight $w_r = w_c N/M$

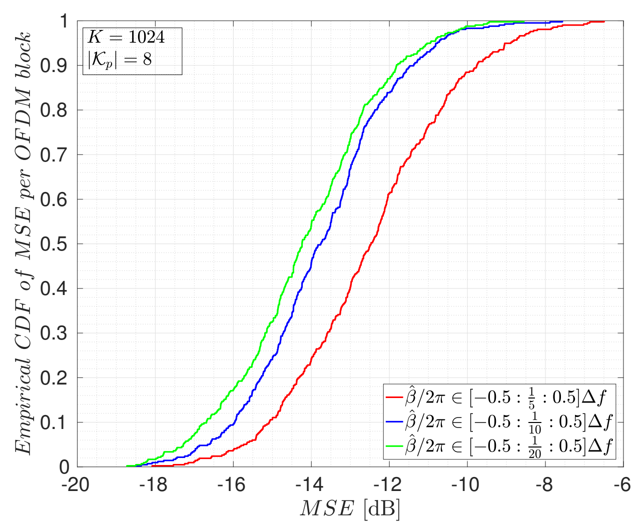


Fig. 3. Estimated CDF of the MSE for hypothesis intervals in S2. The CDFs reflect all 52 transmission with $K = 1024$ carriers during MACE'10.

varies from 3.3 to 30 corresponding to code rates from 0.1 to 0.9 [5]. We use soft decision decoding that takes the likelihood ratio for each code-bit as input. Decoding is performed based on the probability propagation algorithm which can be seen as an instance of the sum-product algorithm [6]. Using the hypothesis interval $[-0.5\Delta f, 0.5\Delta f]$ with steps $\Delta f/10$ and code rate as high as 0.8, we achieve BER as low as 5×10^{-5}. Code rates below 0.8 result in low BER values that cannot be measured with the existing data.

V. Conclusion

We considered differentially coherent detection of acoustic OFDM signals and targeted the frequency offset through a hypothesis testing approach. This simple search technique can be used as a stand-alone approach for differentially coherent detection, but it can also be used as a pre-processing stage

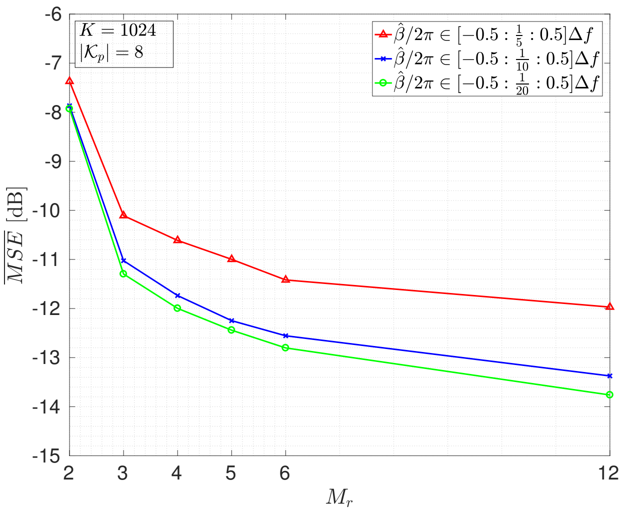


Fig. 4. Average MSE versus the number of receiving elements. The number of carriers and pilots are 1024 and 8, respectively. The receiving elements are maximally-spaced.

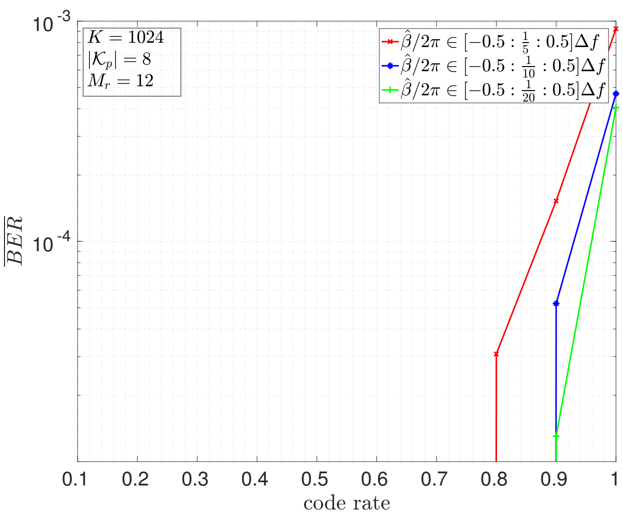


Fig. 5. Average BER versus the rate of the LDPC code. The results reflect all 52 transmissions with 1024 carriers during MACE'10. The proposed frequency synchronization method enables excellent performance with BER $= 5 \times 10^{-5}$ using code rates as high as 0.8.

for coherent detection. Its key feature is that only a few pilots suffice to determine the frequency shift, and once the frequency offset has been compensated, data symbols can be detected either in a coherent or in a differentially coherent manner.

We presented a comprehensive performance analysis using experimental signals recorded over a mobile acoustic channel. Our results show that the proposed method delivers an average MSE below -12 dB for 84% of OFDM blocks and enables a very high rate LDPC code to achieve an excellent BER of 5×10^{-5} at very low computational complexity.

References

[1] Y. M. Aval and M. Stojanovic, "Differentially coherent multichannel detection of acoustic OFDM signals," *IEEE J. Ocean. Eng.*, vol. 40, no. 2, pp. 251–268, Apr. 2015.

[2] K. Tu, T. M. Duman, M. Stojanovic, and J. G. Proakis, "Multiple-resampling receiver design for OFDM over doppler-distorted underwater acoustic channels," *IEEE J. Ocean. Eng.*, vol. 38, no. 2, pp. 333–346, 2013.

[3] B. Li, S. Zhou, M. Stojanovic, L. Freitag, and P. Willett, "Multicarrier communication over underwater acoustic channels with nonuniform doppler shifts," *IEEE J. Ocean. Eng.*, vol. 33, no. 2, pp. 198–209, Apr. 2008.

[4] B. Muquet, Z. Wang, G. B. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefixing or zero padding for wireless multicarrier transmissions?" *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2136–2148, Dec. 2002.

[5] W. Ryan and S. Lin, *Channel codes: classical and modern*. Cambridge University Press, 2009.

[6] D. J. MacKay, "Good error-correcting codes based on very sparse matrices," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 399–431, 1999.