Energy Optimization with Delay Constraints in Underwater Acoustic Networks

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***Abstract*—Underwater Acoustic Sensor Networks find use in critical time-sensitive applications such as disaster prevention and coastline protection. The sensor nodes used in such networks are powered by batteries that are difficult to recharge or replace. Hence it is imperative that routing algorithms used in such networks be very energy efficient while also satisfying necessary delay constraints for time-sensitive applications. Unlike the radio frequency medium, underwater acoustic channels have low bandwidth, large propagation delays and long multipath delay spreads. While energy-efficient routing is an actively researched area for terrestrial radio frequency networks, results from those studies generally do not apply to underwater acoustic networks due to vast differences in channel characteristics. In this paper we explore delay-constrained energy optimization for routing in underwater acoustic sensor networks. Specifically, we propose an offline Mixed Integer Linear Programming based routing algorithm that enables computation of delay constrained energy efficient routes.**

I. Introduction

In an Underwater Acoustic Sensor Network (UWA-SN) an arbitrary number of sensors are geographically distributed under the ocean in a given area to collaboratively collect and relay data to a centralized sink. Such networks are used for time-sensitive applications such as disaster prevention, coastline protection, etc. [1] and for time-insensitive applications such as monitoring of habitat counts, ocean temperature and carbon dioxide levels [2]. These sensors are battery operated. Since they are not easily accessible after deployment, recharging or replacing their batteries is difficult. Hence minimizing energy usage across all layers in the protocol stack becomes essential for such networks. In this paper we will focus on energy efficient routing for time-sensitive applications of UWA-SNs.

Cross layer power minimization in UWA-SNs has been studied in [3], [4], [5], [6] and [7]. While [3] proposes an energy optimized MAC protocol, [4] explores a geographic forwarding routing scheme. Ref. [5] focuses on energy efficient reliable broadcast in UWA-SNs and [7] proposes a focused beam routing protocol. Ref. [6] puts forth an adaptive power allocation technique to minimize energy usage. Further, [8] studies the effect of large propagation delays on the throughput of UWA-SNs. While cross layer energy optimization in UWA-SNs has received much attention from the academic community, the number of routing solutions that can be used for time-sensitive applications, are very limited. Accommodating hard delay constraints significantly complicates the energy optimization problem.

The main contribution of this paper is to use a Mixed Integer Linear Programming (MIP) based optimization framework to compute delay-constrained energy-efficient TDMA routes for wireless UWA-SNs. This paper builds on the convex optimization approach proposed in [9] for Radio Frequency (RF) networks. The channel characteristics of an underwater acoustic medium differ from that of an RF medium in a number of ways. The propagation delay in UWA-SNs is five times larger than in RF networks. Propagation delay plays a major role in the performance of UWA-SNs especially since they cover several square kilometers unlike the small area relays in RF networks. The underwater acoustic channel is also characterized by low bandwidth and significant frequency-selective multipath fading. Due to the large number of variable factors in the underwater acoustic medium, the relationship between power and bit rate is non-linear. This makes applying convex optimization procedures problematic. Hence the solution proposed in [9] cannot be applied directly to UWA-SNs.

We use the water-filling power allocation approach [10] to model the underwater frequency selective multipath fading channel. The complex channel model and the use of water-filling techniques make energy usage computations for underwater acoustic channels analytically intractable. Hence unlike the closed-form convex optimization in [9], determining the optimal delay-constrained energy-efficient routes for UWA-SNs becomes a much harder non-linear non-convex optimization problem. Hence we use an MIP approximated formulation of the problem to pre-compute energy-efficient delay-constrained TDMA routes. We call our scheme Delay-constrained Energy-efficient Routing (DER).

We demonstrate that our proposed method is more energy efficient for UWA-SNs than the Least Cost Routing [7] or the Greedy Geographic Forwarding [4] schemes.

II. System Overview

We consider an underwater network where the sensors are stationary and are geographically distributed in an arbitrary manner within a given area. Each of the sensors produces data at a random rate from a pre-defined range. The sensors

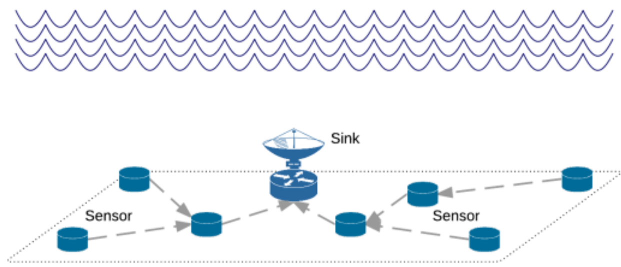

Fig. 1. Underwater Acoustic Sensor Network (UWA-SN)

collaborate amongst themselves to relay each other's traffic to a centralized sink. We assume that there is only one sink in the network. The network uses a slotted synchronous TDMA MAC scheme as described in [9] with a frame length of T seconds which is divided into multiple scheduler time slots of $\triangle$ seconds each. Each of the links in the network is allocated a variable number of time slots. Links are scheduled one at a time in units of $\triangle$ seconds and hence only one link transmits at any point in time thus avoiding interference. Offline routing algorithms are used to pre-calculate the number of time-slot allocations per link and the amount of data transmitted on each of the links. Data sent by the sensors to the sink could get load balanced over multiple outgoing links.

The path loss in an underwater acoustic channel depends on both the signal frequency and the distance [11]. The absorption coefficient in an underwater acoustic channel increases very rapidly with frequency [12]. Given the frequency selective fading nature of the underwater acoustic channel, we use the frequency domain water-filling power allocation method [10] to maximize channel capacity. With a required maximum Bit Error Rate (BER) of 10^{-4}, we generate a look-up table that can be used to determine the transmit power required for a given distance and transmission rate. We use the channel transfer function defined in [11] to determine the SNR for each of the sub-channels. In order to model the limitations of sharing the bandwidth, we assume that all sensors consume a fixed power for processing relay traffic.

Underwater acoustic channels exhibit large propagation delays, relative to RF propagation, owing to the speed of sound in sea water ($c_s \approx 1500 \, \frac{m}{s}$). We factor the propagation delay into our timing constraints. Assuming ideal transmission conditions, we consider fixed packet losses in the network with a packet error rate given by $\left(1-(1-BER)^{Nbits}\right)$, where $Nbits$ is the number of bits in a packet. In our simulation, we also account for absorption and spreading loss, ambient noise, electrical-to-acoustical conversion loss, and additional losses due to random effects such as scattering, Doppler shifting and the effects of movement of sensor nodes due to water currents.

III. Delay-constrained Energy-efficient Routing (DER)

The Delay-constrained Energy-efficient Routing scheme is an offline algorithm that uses an MIP formulation of the energy optimization routing problem to compute delay constrained routes for underwater acoustic TDMA networks. As in [9], we use a constrained optimization strategy. Due to the complex non-linear relationship between the energy consumed, the number of transmitted bits and the transmit time for an underwater acoustic channel, the minimization problem is not convex. DER finds an efficient, global solution by reformulating the problem as an MIP optimization. Additionally, DER incorporates underwater propagation delays to the timing constraints of the problem and also accounts for relay processing power while minimizing the energy consumption.

A. DER Optimization problem formulation

For every sensor, DER selects a fixed set of next hop candidate nodes by picking a given number of nearest neighbors from the list of all nodes that are closer to the sink. Given the geographic coordinates of the sensors, the data generation rate of the nodes and the maximum time period within which data from all sensor nodes is expected to reach the sink, DER computes the optimal set of routes that will use the least amount of energy. DER output includes the selected next hop nodes for all sensors, the time allocated per link and the data transmission rate for each of the links.

Let N be the total number of nodes in the network and p_r be the required relay processing power per sensor. For every sensor node i, let r_i denote the rate at which the node generates data to send, let ψ_i represent the set of next hop candidates that are closer to the sink than i and let $\psi_i^{'}$ represent the set of neighboring nodes for which i is a next hop candidate node. Let $i \rightarrow j$ represent the link between nodes i and j. Then for every link $i \rightarrow j$ between the sensor nodes and their corresponding next hop candidates we use W_{ij}, p_{ij}, $\triangle_{ij}$, τ_{ij} and R_{ij} to represent the number of transmitted bits, transmit power, allocated transmit time, propagation time and the transmit rate, respectively. T is the maximum time allowed for data from all sensors to reach the sink. We use P to denote the relation between p_{ij} and R_{ij}, ie., $p_{ij} = P\left(R_{ij}\right)$.

The solution to the optimization problem given in Eqs. (1)-(3) provides the routes, link rates and link time-allocation that consume the least amount of energy.

$$min \quad \sum_{i=1}^{N-1} \sum_{j \in \psi_i} \left(P\left(\frac{W_{ij}}{\triangle_{ij}}\right) + p_r \right) \triangle_{ij} \tag{1}$$

$$s.t. \quad \sum_{i=1}^{N-1} \sum_{j \in \psi_i} \left(\triangle_{ij} + \tau_{ij}\right) \leq T \tag{2}$$

$$\begin{gathered} \sum_{j \in \psi_i} W_{ij} - \sum_{j \in \psi_i^{'}} W_{ji} = r_i T \\ i = 1, \ldots, N \end{gathered} \tag{3}$$

The minimization objective function (1) is the sum total of energy consumed on every link in the network. The delay constraint (2) specifies an upper bound on the sum total of allocated transmit times and the propagation time of all the links. The data constraint (3) ensures that for every sensor the difference between the amount of outgoing data and incoming data is equal to the amount of data generated by the sensor. The $P\left(\frac{W_{ij}}{\triangle_{ij}}\right)\triangle_{ij}$ term in the above objective function is non-linear and it represents ϵ_{ij}, the energy consumed on link $i \rightarrow j$. The constraints in the above problem are already linear.

The key to linearizing the optimization problem is to substitute the nonlinear term in the objective function with the variable ϵ_{ij} and then add additional linear constraints to establish the relationship between $\log(\epsilon_{ij})$, $\log(W_{ij})$ and $\log(\triangle_{ij})$.

Since $\epsilon_{ij} = P\left(\frac{W_{ij}}{\triangle_{ij}}\right)\triangle_{ij}$, we have that:

$$\log(\epsilon_{ij}) = \log\left(P\left(\frac{W_{ij}}{\triangle_{ij}}\right)\right) + \log(\triangle_{ij}) \tag{4}$$

Let p^L_{ij} and R^L_{ij} represent the logarithm of the transmit power and the transmit rate respectively. Let P_L represent the relation between p^L_{ij} and R^L_{ij}, ie., $p^L_{ij} = P_L\left(R^L_{ij}\right)$. Then we have:

$$\begin{aligned}\log(\epsilon_{ij}) = \; & P_L\left(\log(W_{ij}) - \log(\triangle_{ij})\right) \\ & + \log(\triangle_{ij})\end{aligned} \tag{5}$$

We now use the piecewise linear approximations of P_L and the log functions to formulate our MIP problem. Let $\widetilde{f}(x)$ represent a piecewise linear approximation of the function $f(x)$. Then, DER's MIP optimization problem can be written as

$$min \quad \sum_{i=1}^{N-1} \sum_{j\in\psi_i} (\epsilon_{ij} + p_r\triangle_{ij}) \tag{6}$$

$$s.t. \qquad \sum_{i=1}^{N-1} \sum_{j\in\psi_i} (\triangle_{ij} + \tau_{ij}) \leq T \tag{7}$$

$$\begin{gathered}\sum_{j\in\psi_i} W_{ij} - \sum_{j\in\psi'_i} W_{ji} = r_i T \\ i = 1, ..., N\end{gathered} \tag{8}$$

$$\begin{gathered}R^L_{ij} = \widetilde{\log}(W_{ij}) - \widetilde{\log}(\triangle_{ij}) \\ i = 1, ..., N;\ j \in \psi_i\end{gathered} \tag{9}$$

$$\begin{gathered}\widetilde{\log}(\epsilon_{ij}) = \widetilde{P_L}\left(R^L_{ij}\right) + \widetilde{\log}(\triangle_{ij}) \\ i = 1, ..., N;\ j \in \psi_i\end{gathered} \tag{10}$$

The objective function (6) is the same as (1) but with the non-linear term replaced with the variable ϵ_{ij}. The rate constraint (9) establishes the relationship between the transmit rate, bits transmitted and transmit time variables. The energy constraint (10) establishes the relationship between the energy consumed, transmit power and the transmit time variables. Note that the logarithm of the transmit power is expressed as a function of the transmit rate in the energy constraint. Appendix A explains the implementation of the optimization model in further detail.

As a final step, the per-link transmit time $\triangle_{ij}$ and data size W_{ij} values computed by the MIP optimization are then used to recalculate the actual per-link power usage and the total energy consumed.

IV. Simulation

We compare the performance of DER with a Least Cost Routing (LCR) [7] and a Greedy Geographic Routing (GGR) [4] methods.

A. Least Cost Routing (LCR)

LCR is an offline routing approach that uses the Dijkstra algorithm (as used for comparison in [7]) to compute the least cost route between all nodes in the network to the sink. A finite number of next hop candidate nodes that are closer to the destination, are shortlisted for all the sensor nodes. For each of the next hop candidates, the transmission power required to establish a link with a predefined data transmission rate, is used as the cost for the link. Dijkstra algorithm is then used to calculate the least cost paths from all the sensor nodes in the network to the sink.

B. Greedy Geographic Routing (GGR)

The GGR algorithm that we consider in this paper is a geographic forwarding algorithm described in [4] that has been modified to run offline. The GGR algorithm uses discrete power levels and considers both distance and the required energy as factors for deciding on the next hop node. The sensor nodes select the transmission power to use from a finite set of power levels. The algorithm iterates through all the sensor nodes in descending order of its distance from the sink. For each of the sensor nodes, it shortlists a finite number of next hop candidate nodes that are closer to the destination. The amount of energy required to transmit is then computed for all of the next hop candidates, using the power look-up table, the link distance, the amount of incoming relay traffic and the data generation rate of the sensor. It then selects the next hop which requires the least amount of energy to transmit.

C. Simulation Setup

In the model used for simulation, the sink was located at the center of a two dimensional network. For all three routing algorithms (LCR, GGR and the DER), results were captured for a varying number of sensor nodes in the network. For every test with a given number of sensor nodes, results were averaged over 500 iterations where the location of the sensor nodes were randomly changed with each iteration. Each of the sensors generated data at a rate that was randomly assigned from a predefined range of values. As long as the relay processing power is positive, it's value does not change the simulation results. Hence we use an arbitrary value of 0.05~W for it. For the GGR algorithm 8 power levels, uniformly spaced in a predefined range, were available for the sensors to use. The number of sensors and the average energy per bit in mJ were recorded for each of the tests. Table I shows the values of different network parameters used in the tests.

TABLE I
VALUE OF PARAMETERS USED IN THE SIMULATIONS

Parameter	Value
Network dimensions	10 km x 10 km
Number of sensors	5 to 30
Max outgoing links per node	5
Max time period (T)	70 s
Sensor data generation rate ($\pm 10\%$)	30 bps
Max transmit power	182 dBreμPa
Maximum BER	10^{-4}
Carrier frequency	25 kHz
Bandwidth	5 kHz
Noise power spectral density	55 dBreμPa/Hz
Spreading factor [13]	1.5
Electrical to Acoustical conversion	172 dBWreμPa
Transducer efficiency	90 %
Relay processing power	0.05 W

D. Results

Fig. (2) compares the average energy per bit measurements seen with DER to that measured with the LCR and GGR algorithms. DER solves for the least energy utilizing route sets. While DER achieves global optimization of the total energy used in the network, individual next hop decisions made using the GGR algorithm only use localized information, and hence are only locally optimal. As a result, the DER algorithm is seen to be 2 to 5 dB more energy-efficient than GGR depending on the number of sensors. The amortized energy cost of making sub-optimal routing decisions with LCR is higher for low density than for high density networks. Hence, LCR's performance improves as the number of nodes in the network increases. Although the LCR algorithm picks the least power path from each of the sensors to the sink, energy usage is only optimized per path as opposed to the global optimization performed by DER. DER also uses multiple outgoing links from each of the sensors, and is hence able to distribute the load among all the relays in a more energy-efficient manner. This is reflected in the test results where DER is observed to be 1 to 7 dB more energy-efficient than LCR depending on the number of nodes in the network.

Unlike GGR and LCR, DER guarantees that data from all the sensors will reach the sink within a specified time period. The max time period used in the delay constraints of DER's MIP optimization accommodates the underwater propagation delays as well. With DER, once the load in the network crosses a critical threshold, the total energy usage in the network begins to increase rapidly. This results from the fact that DER satisfies delay constraints by increasing the transmit powers to achieve higher link data transmission rates. As the time period for the DER delay constraints are relaxed, the amount of load the network can handle before it reaches the critical threshold also increases. This is intuitively expected and is illustrated in Fig. (3).

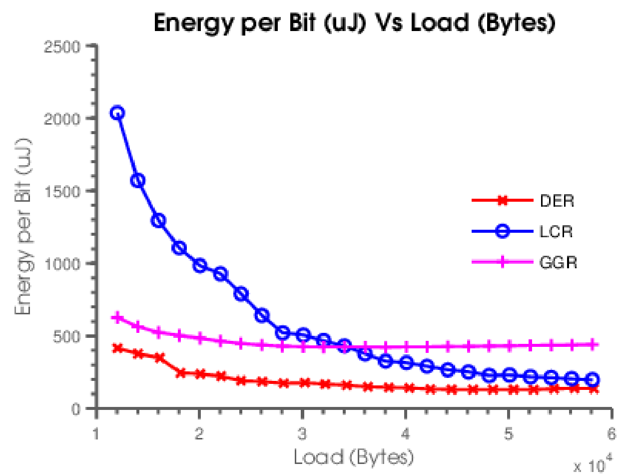

Fig. 2. Plot comparing energy per bit (mJ) for LCR and GGR with DER routing

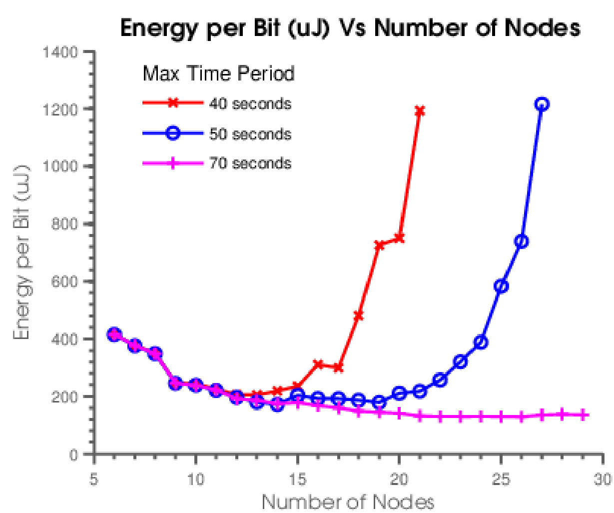

Fig. 3. DER with varying Time Period

Consistent results are also seen by increasing the network load while keeping the time period fixed. As illustrated in Fig.

(4), the threshold number of sensors beyond which the energy usage begins to rapidly increase is observed to be smaller when the amount of load in the network is increased.

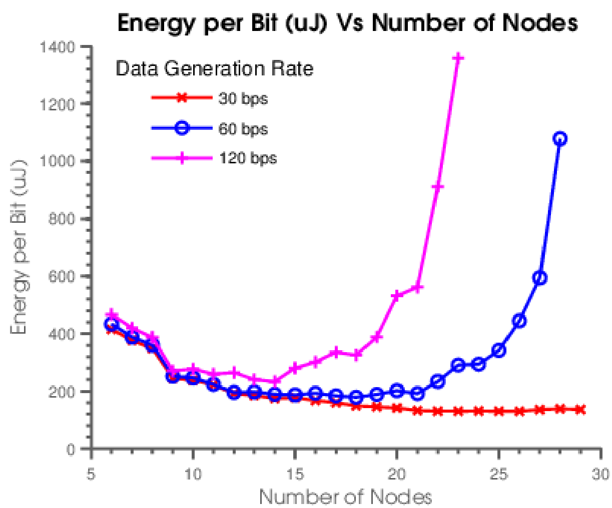

Fig. 4. DER with varying Load

With increasing network area, the average distance between nodes increases, increasing the energy usage per bit as demonstrated in Fig. (5).

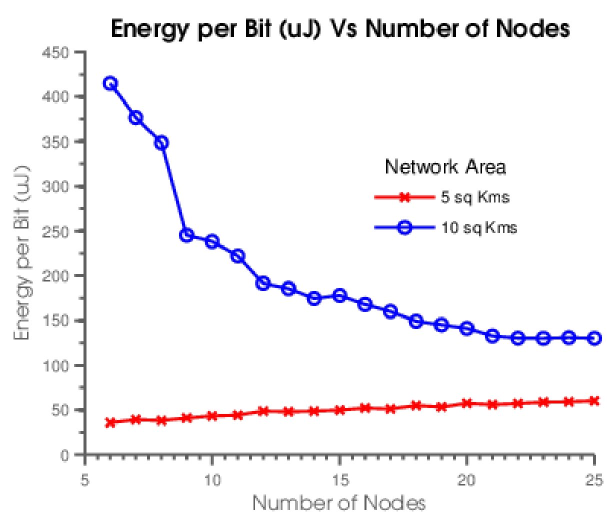

Fig. 5. DER with varying Network area

V. Summary

Disaster prevention and coastal protection are some time-sensitive applications of underwater acoustic sensor networks [1]. Since the batteries used in these sensors are not easily replaceable, it is imperative that the route-sets used in such networks are energy-efficient. Underwater acoustic channel have large propagation delays, limited bandwidth, frequency selective multipath fading and other characteristics that are vastly different from a terrestrial RF channel. As a result, solutions from RF networks cannot be applied directly to UWA-SNs. While a number of papers have looked into energy-efficient routing in UWA-SNs, there are limited number of available solutions that could be used for delay-intolerant applications. In this paper we propose a Delay-constrained Energy-efficient Routing (DER) approach which uses an MIP formulation of the routing problem to compute delay-intolerant energy-efficient route-sets. Depending on the number of nodes in the network, simulation results indicate that our approach is 2 to 5~dB more energy-efficient than the Greedy Geographic Routing algorithm and about 1 to 7~dB more energy efficient than the Least Cost Routing algorithm. Unlike GGR which uses a locally optimal approach and LCR which optimizes one route path at a time, the DER approach results in a route-set that is globally optimal.

DER uses a centralized offline routing approach. The algorithm should be run every time new nodes are added to the network. It could be run at the sink and the solution route-set could then be downloaded to the sensor nodes. In addition to the carrier frequency, bandwidth and the maximum time period, the node running the algorithm needs to know the geographic location and the data generation rate of each of the sensors.

Future work will include TDMA schedule computation based on the optimal route-set and the investigation of a decentralized algorithm to solve the delay-constrained energy optimization problem. Further we will also study DER's performance in networks with multiple sinks and in three dimensional network topology models.

Appendix A
DER Implementation

Here we provide a detailed description of how DER was implemented.

We know that

$$\epsilon_{ij} \quad = \quad p_{ij}\triangle_{ij} \tag{11}$$

Taking the logarithm of both sides, we have that

$$log\left(\epsilon_{ij}\right) \quad = \quad log\left(p_{ij}\right)+log\left(\triangle_{ij}\right) \tag{12}$$

Let $\mathbf{S^{\epsilon}_{ij}}$, $\mathbf{S^{\triangle}_{ij}}$ and $\mathbf{S^{W}_{ij}}$ each be a vector of k Special Ordered Set Type 2 ($SOS2$) variables [14] that are used to represent piecewise linear approximations of $\log\left(\epsilon_{ij}\right)$, $\log\left(\triangle_{ij}\right)$ and $\log\left(W_{ij}\right)$, respectively. Let $\mathbf{L_x}$ and $\mathbf{L_y}$ each be a vector of k $SOS2$ breakpoints (knots and coefficients) for the log function such as Z and $\widetilde{\log}\left(Z\right)$can be derived from the following inner product expressions: $Z=\left\langle \mathbf{L_x},\mathbf{S^Z}\right\rangle$ and $\widetilde{\log}\left(Z\right)\approx\left\langle \mathbf{L_y},\mathbf{S^Z}\right\rangle$. Let $\mathbf{S^{P}_{ij}}$ be a vector of k $SOS2$ variables that are used to represent piecewise linear approximations of P_L. We use a maximum error of 1% for our piecewise linear approximations.

The relationship between p_{ij}^L and p_{ij}^L can now be approximated as follows:

$$p_{ij}^L \approx \left\langle \mathbf{L_y}, \mathbf{S_{ij}^P} \right\rangle \tag{13}$$

$$R_{ij}^L = \left\langle \mathbf{L_x}, \mathbf{S_{ij}^P} \right\rangle \tag{14}$$

We know that the data transmission rate R_{ij} is given by

$$R_{ij} = \frac{W_{ij}}{\triangle_{ij}} \tag{15}$$

Taking the logarithm of both sides of the above expression, we have that

$$R_{ij}^L = \log(W_{ij}) - \log(\triangle_{ij}) \tag{16}$$

Let d_{ij} be the distance between nodes i and j. Then the propagation time for link $i \to j$ is given by

$$\tau_{ij} = d_{ij} c_s \tag{17}$$

The DER MIP optimization problem can now be modeled as follows:

$$min \quad \sum_{i=1}^{N-1} \sum_{j \in \psi_i} \left(\left\langle \mathbf{L_x}, \mathbf{S_{ij}^{\epsilon}} \right\rangle + p_r \triangle_{ij} \right) \tag{18}$$

$$s.t. \qquad \sum_{i=1}^{N-1} \sum_{j \in \psi_i} \left(\left\langle \mathbf{L_x}, \mathbf{S_{ij}^{\triangle}} \right\rangle + \tau_{ij} \right) \leq T \tag{19}$$

$$\begin{gathered} \sum_{j \in \psi_i} \left\langle \mathbf{L_x}, \mathbf{S_{ij}^W} \right\rangle - \sum_{j \in \psi_i'} \left\langle \mathbf{L_x}, \mathbf{S_{ji}^W} \right\rangle \\ = r_i T \\ i = 1, ..., N \end{gathered} \tag{20}$$

$$\begin{gathered} \left\langle \mathbf{L_y}, \mathbf{S_{ij}^{\epsilon}} \right\rangle = \left\langle \mathbf{L_y}, \mathbf{S_{ij}^P} \right\rangle + \left\langle \mathbf{L_y}, \mathbf{S_{ij}^{\triangle}} \right\rangle \\ i = 1, ..., N; \; j \in \psi_i \end{gathered} \tag{21}$$

$$\begin{gathered} \left\langle \overrightarrow{\mathbf{1}}, \mathbf{S_{ij}^{\epsilon}} \right\rangle = O_{ij} \\ \left\langle \overrightarrow{\mathbf{1}}, \mathbf{S_{ij}^P} \right\rangle = O_{ij} \\ \left\langle \overrightarrow{\mathbf{1}}, \mathbf{S_{ij}^W} \right\rangle = O_{ij} \\ \left\langle \overrightarrow{\mathbf{1}}, \mathbf{S_{ij}^{\triangle}} \right\rangle = O_{ij} \\ i = 1, ..., N; \; j \in \psi_i \end{gathered} \tag{22}$$

Except for the $SOS2$ variables and O_{ij}, all other variables in the above optimization problem can assume real values. The minimization objective function (18) is the sum total of energy consumed on every link in the network. The above delay constraint (19) incorporates the propagation delay for every link.The data constraint (20) ensures that for every sensor the difference between the amount of outgoing data and incoming data is equal to the amount of data generated by the sensor. The energy constraint (21) incorporates piecewise linear approximations of the $\log$ functions into Eq. (12). Finally, the constraints (22) were added for all the $SOS2$ variable sets for every link, where $\overrightarrow{\mathbf{1}}$ is an all ones vector and O_{ij} is a binary decision variable that indicates whether the link between nodes i and j is used in the solution route-set. For links that are not used, the above constraints ensure that all the variables associated with that link get assigned a value of 0.

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