# Analysis of the Impact of Channel Estimation Errors on the Performance of a Decision-Feedback Equalizer in Fading Multipath Channels

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*Abstract*— **A coherent receiver with a decision-feedback equalizer (DFE) operating on a Rayleigh fading channel under a suitable adaptive algorithm is considered. In the analysis of a DFE, a common assumption is that the receiver has perfect knowledge of the channel impulse response. However, this is not the case in practice, and for a rapidly fading channel, errors in channel tracking can become significant. We analyze theoretically the impact of these errors on the performance of a multichannel DFE. The expressions obtained for the achievable average MPSK bit error probabilities depend on the estimation error covariance. In order to specify this matrix, we focus on a special case when a Kalman filter is used as an optimal channel estimator. In this case, the probability of bit error can be assessed directly in terms of channel fading model parameters, the most interesting of which is the fading rate. Our results show the penalty imposed by imperfect channel estimation, as well as the fading-induced irreducible error rates.**

## I. INTRODUCTION

Due to its low computational complexity and near-optimal performance, the decision-feedback equalizer (DFE), operating under a suitable adaptive algorithm, is used in a variety of time-dispersive fading channels, such as a mobile radio channel, a troposcatter channel, or an underwater acoustic channel. It is well-known that the critical issue for its performance in a rapidly changing channel is the tracking capability of the underlying adaptive algorithm. Although a vast body of literature has been devoted to the performance analysis of the DFE, it seems that none addresses the impact of fading-induced imperfect channel tracking. The goal of this analysis is to determine the performance limitations of a coherent receiver which incorporates a DFE, under the conditions that only the statistical properties of the channel are known to the receiver.

The primary factor which causes the degradation of the DFE performance from the matched filter bound is the residual intersymbol interference (ISI) from future symbols. Its impact on the average achievable bit error probability has been theoretically analyzed in [1], assuming that a complete side information about the channel state is available. In practice, however, the channel is not known, and in the case of a rapidly changing environment, it has to be tracked continuously. Errors in channel estimation will result in additional DFE performance degradation, which is quantitatively analyzed in this paper.

We assume that the tap adjustment of the equalizer is performed indirectly via a channel estimate. Such an approach was found in [2], [3] to be more robust with respect to channel fading than the classical direct adaptation of the DFE taps. The equalizer itself can be expected to track the channel phase fluctuations, provided that it has been relieved of the tap rotation problem by a preceding phase tracking loop. The channel estimator accomplishes planar carrier phase tracking, thus providing coherent demodulation. The residual error in carrier phase tracking can then be incorporated into the overall channel estimation error, which is the approach taken in [4]. However, only the case of a nondispersive channel is treated in [4]. The impact of channel estimation errors on the performance of a maximum likelihood sequence estimator on a frequency selective channel was analyzed in [5] for the case of a slowly fading channel.

We obtain the expression for the average probability of error in terms of the channel estimation error covariance matrix, which in turn depends on the particular channel estimation technique used. A special case of a Gauss-Markov channel model, accompanied by a Kalman filter as a channel estimator, is considered. In this case, the probability of error can be assessed in terms of channel model parameters, e.g., fading rate. Besides being generally accepted for a variety of radio communication channels, such a model was also used in [6] to describe the underwater acoustic channel fluctuations. Other estimation methods, namely least squares and correlation techniques, have been addressed in [3], [5]. The performance of a DFE aided by a Kalman filter as a channel estimator, and related channel modeling issues were addressed through simulation in [7].

After discussing the general channel and system model in Section II, in Section III we derive the optimal values of the DFE parameters, subject to the fact that only the

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channel estimates are available, and using the minimum mean-squared (MMSE) criterion. In Section IV, the analysis of the average bit error probability which takes into account the channel estimation errors is presented. We consider a general case of diversity reception and MPSK modulation format. Section V deals with the computation of the needed error covariance matrices when the Kalman filter is used for channel tracking. Finally, in Section VI the results obtained are illustrated through several numerical examples.

## II. Channel and System Model

The block diagram of the transmitter, the channel and the receiver is shown in Fig.1. The transmitted sequence

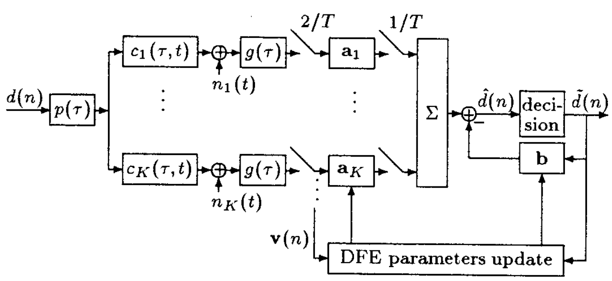


Fig. 1. System block diagram.

of data symbols is denoted by $\{d(n)\}$, and any linear modulation format is applicable. The signal at the transmitter is shaped by a filter having an impulse response $p(\tau)$[1] and transmitted over $K$ diversity channels with impulse responses given by $\{c_k(\tau,t)\}$ as a function of the delay $\tau$ at time $t$. The filters at the receiver front end have the impulse response $g(\tau)$. The transmitter and receiver filters are often chosen to have a square root Nyquist characteristic, so that the overall response $f(\tau)=p(\tau)*g(\tau)$ has a Nyquist characteristic, resulting in no ISI if the channel is ideal. However, it is not necessary to do so when the channel introduces time-dispersion. When the channel response is not known, the receiving filter is often chosen to have a rectangular low-pass transfer function.

The independent AWGN terms $\{n_k(t)\}$ are zero-mean with power spectral density $N_0$. The total received signal power is $E_s/T$, $T$ being the signaling interval. The received signal, as seen by the $k^{th}$ equalizer, is given by

$$v_k(t)=\sum_n d(n)h_k(t-nT,t)+\nu_k(t) \quad (1)$$

where $h_k(\tau,t)=c_k(\tau,t)*f(\tau)$ is the overall channel response at time $t$, and the additive noise $\nu_k(t)$ is zero-mean Gaussian with

$$E\{\nu_k(t)\nu_k^*(t-\tau)\}=N_0G(\tau). \quad (2)$$

[1] All the signals are represented in equivalent complex baseband form.

The feedforward equalizers are fractionally spaced for purposes of obtaining adaptive matched filtering and correct symbol timing. For signals band-limited to $1/T$, a fractional spacing of $T/2$ is assumed without loss of generality. The total time-span of the $N$-tap feedforward equalizer needs to be at least as long as that of the the significant part of the channel response. The $M$ feedback filter taps are assumed to cover all of the ISI resulting from the previously detected symbols. The outputs of the feedforward equalizers are produced once per symbol interval, and they are given by[2]

$$\begin{aligned} x_k(n) &= [a^*_{k,-N_1}\cdots a^*_{k,N_2}]\begin{bmatrix} v_k(nT+N_1T/2)\\ \vdots \\ v_k(nT-N_2T/2)\end{bmatrix}\\ &= \mathbf{a}'_k\mathbf{v}_k(n). \end{aligned} \quad (3)$$

The vector $\mathbf{v}_k(n)$ of received signal samples in the $k^{th}$ diversity channel can be represented as

$$\mathbf{v}_k(n)=\sum_m d(m)\mathbf{h}_k(n-m,n)+\boldsymbol{\nu}_k(n) \quad (4)$$

where

$$\mathbf{h}_k(m,n)=\begin{bmatrix} h_k(mT+N_1T/2,nT+N_1T/2)\\ \vdots\\ h_k(mT,nT)\\ \vdots\\ h_k(mT-N_2T/2,nT-N_2T/2)\end{bmatrix} \quad (5)$$

is the vector of $T/2$ spaced samples of the overall channel impulse response, shifted by $m$ symbols with respect to the 'centered' channel vector $\mathbf{h}_k(0,n)$, and $\boldsymbol{\nu}_k(n)$ is the vector of $T/2$ spaced noise samples. The coherent combination of the feedforward equalizers outputs is

$$x(n)=\sum_{k=1}^{K}x_k(n)=[\mathbf{a}'_1\ldots\mathbf{a}'_K]\begin{bmatrix}\mathbf{v}_1(n)\\ \vdots\\ \mathbf{v}_K(n)\end{bmatrix}=\mathbf{a}'\mathbf{v}(n). \quad (6)$$

If the composite vector of all the channel responses is formed as

$$\mathbf{h}'(m,n)=[\mathbf{h}'_1(m,n)\ldots\mathbf{h}'_K(m,n)] \quad (7)$$

and similar composition $\boldsymbol{\nu}(n)$ of the noise vectors is made, the composite signal vector $\mathbf{v}(n)$ can be expressed as

$$\mathbf{v}(n)=\sum_m d(m)\mathbf{h}(n-m,n)+\boldsymbol{\nu}(n). \quad (8)$$

Since the composite signal vector can be expressed in the same form as the corresponding single-channel received signal vector (4), the analysis of a multichannel DFE is essentially the same as that of a single-channel DFE.

[2] Prime denotes conjugate transpose, and the equalizer coefficients are taken conjugate for convenience of notation.

## III. DFE Parameter Optimization

In the case of a rapidly time-varying channel, the adaptation of the equalizer tap-weights has to be carried out continuously. There are two ways in which this task can be accomplished. One is the direct adaptation of the equalizer tap coefficients, and the other is their computation from a separately obtained channel estimate. The second approach has the advantage that it is potentially more robust to the channel time-variations [2], and also less computationally complex [3].

So, let us assume that the equalizer adaptation is carried out via a channel estimation process. Let $\hat{\mathbf{h}}(m,n)$ denote the estimate of $\mathbf{h}(m,n)$, the $m^{th}$ shift of the channel vector at time $n$. The corresponding estimation error is defined as

$$\boldsymbol{\varepsilon}(m,n) = \mathbf{h}(m,n) - \hat{\mathbf{h}}(m,n). \tag{9}$$

For the time being, the exact way in which the estimation is performed is not important. We are only concerned with the fact that if the channel is represented as a Gaussian process, as it is the case for Rayleigh fading, we shall assume that its estimate is also a Gaussian process, as well as the estimation error. Even in the case of a fixed, but unknown channel, the Gaussian assumption for the channel estimate would hold due to the Gaussianity of the measurement noise present in the channel estimation. We shall also use the fact that the error is zero-mean and orthogonal to the channel estimate, which establishes the relationship

$$\begin{aligned} & E\{\mathbf{h}(0,n)\hat{\mathbf{h}}'(0,n)\} = E\{\hat{\mathbf{h}}(0,n)\hat{\mathbf{h}}'(0,n)\} \\ = \; & E\{\mathbf{h}(0,n)\mathbf{h}'(0,n)\} - E\{\boldsymbol{\varepsilon}(0,n)\boldsymbol{\varepsilon}'(0,n)\}. \end{aligned} \tag{10}$$

The equalizer tap-weights are chosen to minimize the mean-squared error at the input to the decision device, assuming that the ISI due to previously detected symbols has been completely canceled by the feedback section, and subject to the fact that only the knowledge of the channel estimates and channel statistics is available. In practice, incorrect symbol decisions will affect the performance of both the equalizer and the channel estimator in the decision-directed mode of operation. The induced symbol error propagation then has to be limited by periodically inserting the training sequences into the data stream. Assuming that reliable operation has been achieved in this way, we neglect the effect of symbol errors in what follows.

The output $x(n)$ of the feedforward section is given as

$$x(n) = \sum_m \mathbf{a}'(n)\mathbf{h}(m,n)d(n-m) + \mathbf{a}'(n)\boldsymbol{\nu}(n) \tag{11}$$

and the output of the feedback section of the equalizer is given by

$$y(n) = \sum_{m>0} b_m^*(n)\tilde{d}(n-m) \tag{12}$$

where $\tilde{d}(n)$ denotes the decision made on the symbol $d(n)$. The input to the decision device is then

$$\begin{aligned} \hat{d}(n) \; = \; & \mathbf{a}'(n)\mathbf{h}(0,n)d(n) + \sum_{m\neq 0} \mathbf{a}'(n)\mathbf{h}(m,n)d(n-m) \\ & + \mathbf{a}'(n)\boldsymbol{\nu}(n) - \sum_{m>0} b_m^*(n)\tilde{d}(n-m). \end{aligned} \tag{13}$$

When the decisions are correct with high probability, it is reasonable to assume [1] that the feedback filter completely cancels the ISI due to past symbols. In an ideal DFE this is accomplished when

$$b_{m_{opt}}^*(n) = \mathbf{a}'(n)\mathbf{h}(m,n), \; m = 1,\ldots,M. \tag{14}$$

In a more realistic situation, when only the channel estimates are available, the feedback taps of the DFE will be chosen as

$$b_m^*(n) = \mathbf{a}'(n)\hat{\mathbf{h}}(m,n), \; m = 1,\ldots,M. \tag{15}$$

Assuming correct decisions, and using the feedback coefficients from (15), the decision variable becomes

$$\begin{aligned} \hat{d}(n) \; & = \; \mathbf{a}'(n)[\mathbf{h}(0,n)d(n) + \sum_{m<0} \mathbf{h}(m,n)d(n-m) \\ & \quad + \sum_{m>0} \boldsymbol{\varepsilon}(m,n)d(n-m) + \boldsymbol{\nu}(n)] \\ & = \; \mathbf{a}'(n)[\mathbf{h}(0,n)d(n) + \mathbf{h}^-(n) \\ & \quad + \boldsymbol{\varepsilon}^+(n) + \boldsymbol{\nu}(n)] \end{aligned} \tag{16}$$

The term $\mathbf{h}^-(n)$ represents noise due to the residual ISI. The fact that the summation is carried only over negative values of $m$, indicates that residual ISI comes from future symbols only. The noise term $\boldsymbol{\varepsilon}^+(n)$ results from the channel estimation errors in the feedback taps of the equalizer. Hence, the feedback filter cancels the ISI from past symbols at the expense of introducing this term. Note that if only a linear equalizer were used, the term $\boldsymbol{\varepsilon}^+(n)$ would be replaced by an adequately defined term $\mathbf{h}^+(n)$.

The optimization of the feedforward filter through minimization of the mean-squared error, $MSE = E\{|d(n) - \hat{d}(n)|^2\}$, is made difficult by the fact that the noise terms are data-dependent. In order to avoid this dependence on the unknown symbols, we follow the approach taken in [1], in which the needed covariances of the data-dependent terms are substituted by their expected values. The validity of such approximation was justified in [1] through simulation and measurements on experimental data.

For the case of independent, unit-variance data symbols, the MMSE solution for the feedforward equalizer coefficients is obtained as

$$\mathbf{a}(n) = [\hat{\mathbf{h}}(0,n)\hat{\mathbf{h}}'(0,n) + \mathbf{R}]^{-1}\hat{\mathbf{h}}(0,n) \tag{17}$$

where

$$\mathbf{R} = Cov[\boldsymbol{\varepsilon}(0,n) + \mathbf{h}^-(n) + \boldsymbol{\varepsilon}^+(n) + \boldsymbol{\nu}(n)] \tag{18}$$

is the covariance matrix of the equivalent noise.

Due to the independence among the data symbols, the components of the equivalent noise are assumed to be independent. The covariance of the residual ISI noise term is computed as

$$Cov[\mathbf{h}^-(n)] = \alpha \sum_{m<0} E\{\mathbf{h}(m,n)\mathbf{h}'(m,n)\} \tag{19}$$

where the scale factor $\alpha$ was introduced in [1] to account for averaging over the data sequence. For $E\{|d(n)|^2\} = 1$, it should be taken less than one, so as not to obtain too pessimistic results. The value $\alpha = 0.25$ was found in [1] to give excellent results. Averaging over the data sequence alone leaves random channel components in the matrix $\mathbf{R}$, and the resulting non-Gaussian distribution of the optimal equalizer vector prohibits comprehensive probability of error analysis. However, since most of the symbol errors result from the channel fading, random components of the matrix $\mathbf{R}$ can be substituted by their statistical averages without significantly altering the average probability of error [1]. Hence the form of the expression (19) where averaging is performed over the channel components.

The noise term $\boldsymbol{\varepsilon}^+(n)$ exhibits similar data-dependence as does the term $\mathbf{h}^-(n)$. Therefore, it is reasonable to treat this noise term using the same approximation with averaging over the data sequence, as it was done with the residual ISI noise. Thus, we use

$$Cov[\boldsymbol{\varepsilon}^+(n)] = \alpha \sum_{m>0} E\{\boldsymbol{\varepsilon}(m,n)\boldsymbol{\varepsilon}'(m,n)\}. \quad (20)$$

Since the contribution of this term to the overall noise covariance is many times smaller than that of the residual ISI noise term, the use of the factor $\alpha$ is optional.

If we denote the covariances

$$\begin{aligned} E\{\mathbf{h}(m,n)\mathbf{h}'(m,n)\} &= \mathbf{H}(m) \\ E\{\boldsymbol{\varepsilon}(m,n)\boldsymbol{\varepsilon}'(m,n)\} &= \mathbf{E}(m) \\ Cov[\boldsymbol{\nu}(n)] &= N_0\mathbf{G} \end{aligned} \quad (21)$$

where $\mathbf{G}$ is adequately defined through the receiver filter autocorrelation function $G(\tau)$, the resulting matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \mathbf{E}(0) + \alpha \sum_{m<0} \mathbf{H}(m) + \alpha \sum_{m>0} \mathbf{E}(m) + N_0\mathbf{G}. \quad (22)$$

Assuming that the statistical properties of the channel, as given by the matrix $\mathbf{R}$, are known, the feedforward equalizer taps can be determined from (17). Equivalently, the feedforward filter tap-vector can be written as

$$\mathbf{a}(n) = \frac{1}{1 + \hat{\mathbf{h}}'(0,n)\mathbf{R}^{-1}\hat{\mathbf{h}}(0,n)}\mathbf{R}^{-1}\hat{\mathbf{h}}(0,n). \quad (23)$$

Since the scaling factor in the last expression varies more slowly than the channel estimate $\hat{\mathbf{h}}(0,n)$ [5], and scaling by a positive constant does not affect the performance, the feedforward filter taps can, as well, be determined from

$$\mathbf{a}(n) = \mathbf{R}^{-1}\hat{\mathbf{h}}(0,n). \quad (24)$$

This choice of filter tap-weights, suitable for the subsequent probability of error analysis, is the one that maximizes the signal to noise ratio

$$\overline{SNR} = \frac{|\mathbf{a}'(n)\hat{\mathbf{h}}(0,n)|^2}{\mathbf{a}'(n)\mathbf{R}\mathbf{a}(n)}. \quad (25)$$

Note that due to the presence of channel estimation errors this ratio differs from the true signal to noise ratio which is given by

$$SNR = \frac{|\mathbf{a}'(n)\mathbf{h}(0,n)|^2}{\mathbf{a}'(n)Cov[\mathbf{h}^-(n) + \boldsymbol{\varepsilon}^+(n) + \boldsymbol{\nu}(n)]\mathbf{a}(n)}. \quad (26)$$

The fact that the matrix $\mathbf{R}$ incorporates the estimation error related terms accounts for the best achievable MSE performance when only the channel estimates are available. Nevertheless, the resulting $SNR$ is always lower than in the absence of estimation errors, which will affect the achievable probability of error.

## IV. Probability of Error Analysis with Imperfect Channel Estimation

The probability of bit error for the case of BPSK signaling is given by

$$P_{e2} = P\{Re[\hat{d}] < 0 \mid d = 1\}. \quad (27)$$

Due to the presence of estimation errors, the approach taken in [1] which uses averaging of the conditional probability of error over all channel realizations is no longer applicable, since the resulting $SNR$ is no longer a Gaussian quadratic form of the channel vector $\mathbf{h}(0,n)$. Instead, the expression for the probability of error can be obtained through the following representation which uses a different Gaussian quadratic form. By grouping all the noise terms as

$$\boldsymbol{\xi}(n) = \mathbf{h}^-(n) + \boldsymbol{\varepsilon}^+(n) + \boldsymbol{\nu}(n) \quad (28)$$

and dropping all the indices for simplicity of notation, the estimated data symbol is given by

$$\hat{d} = \mathbf{a}'[d\mathbf{h} + \boldsymbol{\xi}] \quad (29)$$

and the expression (27) can be rewritten as

$$\begin{aligned} P_{e2} &= P\left\{[\ \mathbf{a}' \quad (\mathbf{h}+\boldsymbol{\xi})'\ ]\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{a} \\ \mathbf{h}+\boldsymbol{\xi} \end{bmatrix} < 0\right\} \\ &= P\{Q < 0\} \end{aligned} \quad (30)$$

where $\mathbf{I}$ denotes the identity matrix, and $\mathbf{0}$ denotes the zero matrix of adequate dimensions $KN \times KN$ ($N = N_1 + N_2 + 1$ is the number of feedforward taps in each diversity branch). Under the Gaussian assumption for all the noise terms, and with the equalizer vector proportional to the Gaussian-distributed channel estimate, the decision variable $Q$ in the last expression is a Gaussian quadratic form. Its probability density function is easily obtained [8] once the covariance matrix of the underlying Gaussian vector $[\ \mathbf{a}' \quad (\mathbf{h}+\boldsymbol{\xi})'\ ]$ is known. The resulting probability of error will depend only on the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} Cov\begin{bmatrix} \mathbf{a} \\ \mathbf{h}+\boldsymbol{\xi} \end{bmatrix}. \quad (31)$$

If the $I$ distinct eigenvalues of this matrix are denoted by $\{\lambda_i\}$, and each is of multiplicity $K_i$, the probability of error is given by

$$P_{e2} = \sum_{i:\lambda_i<0} \sum_{k=1}^{K_i} A_{i,k} \quad (32)$$

where $\{A_{i,k}\}$ are the coefficients of the partial fraction expansion of the characteristic function $C_Q(j\omega)$ of the decision variable $Q$,

$$C_Q(j\omega) = \prod_{i=1}^{I} \frac{1}{(1-j\omega\lambda_i)^{K_i}} = \sum_{i=1}^{I}\sum_{k=1}^{K_i} \frac{A_{i,k}}{(1-j\omega\lambda_i)^k}. \quad (33)$$

To denote this dependence on the eigenvalues, we use the shorthand notation

$$P_{e2} = P[\mathbf{A}]. \quad (34)$$

When all the eigenvalues of $\mathbf{A}$ are distinct, the probability of error is simply given by

$$P_{e2} = \sum_{i:\lambda_i<0} A_i, \; A_i = \prod_{j\neq i} \frac{1}{1-\frac{\lambda_j}{\lambda_i}}. \quad (35)$$

In order to find the relevant submatrices comprising the matrix $\mathbf{A}$, we invoke the orthogonality assumptions (10). Using these expressions, and the feedforward filter coefficients from (24), the matrix $\mathbf{A}$ which determines the probability of error (34) is obtained as

$$\mathbf{A} = \begin{bmatrix} [\mathbf{H}(0)-\mathbf{E}(0)]\mathbf{R}^{-1} & \mathbf{R}^{-1}[\mathbf{H}(0)-\mathbf{E}(0)]\mathbf{R}^{-1} \\ \mathbf{R}+\mathbf{H}(0)-\mathbf{E}(0) & \mathbf{R}^{-1}[\mathbf{H}(0)-\mathbf{E}(0)] \end{bmatrix} \quad (36)$$

with $\mathbf{R}$ given in (22).

The impact of estimation errors is reflected in the probability of error through the matrix $\mathbf{A}$ in two ways. First, the signal power, as represented by the matrix $\mathbf{H}(0)$, is reduced by the amount $\mathbf{E}(0)$, and second, the overall noise covariance is increased by the estimation error related terms introduced by the DFE. Note that although the input noise power $N_0$ is shown explicitly only in the thermal noise component of $\mathbf{R}$, the error covariance matrices $\mathbf{E}(m)$ are also affected by this noise through the estimation process. If the overall noise covariance were computed by leaving out the $\mathbf{h}^-(n)$, $\boldsymbol{\varepsilon}^+(n)$ terms, the resulting bit error probability would correspond to the matched filter bound which takes into account the channel estimation errors. Any equalizer will therefore suffer the effects introduced by the term $\mathbf{E}(0)$.

The result obtained is quite general in the sense that it accommodates correlated fading both between the diversity channels and within each of the channels. However, the diversity channels are commonly chosen to be independently fading and we verify our result in the case of independently fading channels with identical statistics. Setting $\mathbf{E}(m) = \mathbf{0}$ in this case, leads to the same result for the bit error probability as the one obtained in [1]. Next, we set the ISI to zero, i.e. disconnect the feedback equalizer, and reduce the feedforward filter to one tap only, assuming that the receiver filter is matched to the transmitter filter and that perfect timing exists. Such configuration corresponds to carrier synchronization using planar filtering, which has been treated in [4]. In this case, for equal energy ($E_s/K$ per channel) independent diversity, the expression for $P_{e2}$ can be obtained in closed form, since there are only two distinct eigenvalues of the matrix $\mathbf{A}$. It is given by

$$P_{e2} = \frac{1}{2}(1-\mu_e)\sum_{k=0}^{K-1}\binom{K-1+k}{k}\left(\frac{1+\mu_e}{2}\right)^k \quad (37)$$

where

$$\mu_e = \sqrt{\frac{\frac{E_s}{KN_0}-\frac{E_e}{KN_0}}{\frac{E_s}{KN_0}+1}} \quad (38)$$

and $E_e/K$ is the variance of the (single) channel tap estimation error in each of the diversity branches. The expression obtained has the same form as the well-known result from [9], except for the fact that $\mu_e$ takes into account the effect of estimation errors. For $K = 1$, the expression (37) reduces to the result from [4].

For QPSK signaling, the probability of bit error, assuming Gray coding, is given by

$$P_{e4} = P\{Re[\hat{d}] < 0 | d = e^{j\pi/4}\} \quad (39)$$

and for 8PSK signaling it is

$$\begin{aligned} P_{e8} &= \frac{2}{3}P\{Re[\hat{d}] < 0 | d = e^{j3\pi/8}\} \\ &+\frac{2}{3}P\{Re[\hat{d}] > 0 | d = e^{j3\pi/8}\} \\ &\cdot P\{Im[\hat{d}] < 0 | d = e^{j3\pi/8}\}. \end{aligned} \quad (40)$$

In order to develop these expressions it is useful to evaluate the general term

$$P\{Re[\hat{d}e^{j\phi_1}] < 0 | d = e^{j\phi_2}\}. \quad (41)$$

Defining

$$\mathbf{A}(\phi) = \begin{bmatrix} [\mathbf{H}(0)-\mathbf{E}(0)]\mathbf{R}^{-1}e^{j\phi} & \mathbf{R}^{-1}[\mathbf{H}(0)-\mathbf{E}(0)]\mathbf{R}^{-1} \\ \mathbf{R}+\mathbf{H}(0)-\mathbf{E}(0) & \mathbf{R}^{-1}[\mathbf{H}(0)-\mathbf{E}(0)]e^{-j\phi} \end{bmatrix} \quad (42)$$

the probability (41) is seen to be equal to $P[\mathbf{A}(\phi_1+\phi_2)]$. Substituting this result into the expressions (39) and (40), we obtain the desired probabilities of error as

$$P_{e4} = P[\mathbf{A}(\pi/4)] \quad (43)$$

and

$$P_{e8} = \frac{2}{3}\{P[\mathbf{A}(3\pi/8)] + (1-P[\mathbf{A}(3\pi/8)])P[\mathbf{A}(\pi/8)]\}. \quad (44)$$

Similarly as for the BPSK case, it can be shown that in the absence of ISI, the terms $P[\mathbf{A}(\phi)]$ reduce to the form (37) with $\mu_e$ being replaced by

$$\mu_e(\phi) = \sqrt{\frac{cos^2\phi(\frac{E_s}{KN_0}-\frac{E_e}{KN_0})}{cos^2\phi\frac{E_s}{KN_0}+sin^2\phi\frac{E_e}{KN_0}+1}} \quad (45)$$

which, when substituted into (43), (44) for $K = 1$ agrees with [4].

## V. Computation of the Kalman Filter Error Covariance Matrices

The question that remains to be discussed is the determination of the error covariance matrix needed for the computation of the probability of error. The value of this matrix obviously depends on the particular technique used for channel estimation. We examine the case when a Kalman filter is used for channel tracking. This is the case of particular interest since the Kalman filter represents the optimal MMSE estimator of the channel process which obeys a Gauss-Markov model, provided the knowledge of the model parameters and given the set of the observed received signal samples.

In practice, the use of a Kalman filter is just a possible choice. Frequently, the channel model is not known and a different estimation procedure is preferred. The performance of a DFE coupled with an optimally designed LMS or RLS channel estimator was analyzed in [3]. On the other hand, since for a variety of channels the model parameters can accurately be obtained from probing measurements [7], we shall assume the optimal regime for the Kalman filter. Although the optimality is conditioned on correct modeling and access to correctly reconstructed past data, these assumptions are expected to bear much less influence on the estimated average performance than does the assumption of a perfectly known channel response.

The overall channel vector which contains all the relevant (nonzero) samples of the channel response at time $nT$, is given as

$$\mathbf{h}'(n) = [\mathbf{h}_1'(n) \dots \mathbf{h}_K'(n)] \tag{46}$$

where

$$\mathbf{h}_k(n) = \begin{bmatrix} h_k(M_1T, nT) \\ h_k(M_1T - T/2, nT) \\ \dots \\ h_k(-M_2T, nT) \\ h_k(-M_2T - T/2, nT) \end{bmatrix} \tag{47}$$

is the $k^{th}$ channel vector at time $n$. The fading channel dynamics are often described by a Gauss-Markov model, which represents the channel vector $\mathbf{h}(n)$ as an autoregressive process in noise [4], [6]. The underlying state-space channel description is given by the process equations

$$\begin{aligned} \mathbf{c}(n+1) &= \mathbf{F}(n+1,n)\mathbf{c}(n) + \boldsymbol{\chi}(n) \\ \mathbf{h}(n) &= \mathbf{T}\mathbf{c}(n) \end{aligned} \tag{48}$$

where $\mathbf{F}(n+1,n)$ is the one-step channel state transition matrix at time $n$, and $\boldsymbol{\chi}(n)$ is the process noise, Gaussian-distributed with zero-mean in the case of Rayleigh fading. For the well-known case of a wide-sense stationary channel, the transition matrix does not depend on time, $\mathbf{F}(n+m,n) = \mathbf{F}(m)$.

The channel states can be thought of as gains of the physical propagation paths. A simple first-order model in which the paths of each channel are independently fading at the same rate is given by

$$\mathbf{c}_k(n+1) = f\mathbf{c}_k(n) + \boldsymbol{\chi}_k(n), \; k = 1, \dots K \tag{49}$$

where the state covariance matrix

$$E\{\mathbf{c}_k(n)\mathbf{c}_k'(n)\} = \frac{1}{1-|f|^2} E\{\boldsymbol{\chi}_k(n)\boldsymbol{\chi}_k'(n)\} \tag{50}$$

is diagonal, and its elements represent the sampled multipath intensity profile of the channel.

The output transformation $\mathbf{T}$ accounts both for the filtering introduced by the physical channel, and the particular transmitter and receiver filtering. Note that due to the receiver filtering and Nyquist rate sampling, the uncorrelated scattering between the propagation paths does not imply the independence of channel vector components.

In the case of independently fading channels, the channel covariance $\mathbf{H} = E\{\mathbf{h}(n)\mathbf{h}'(n)\}$ is block diagonal, with each block corresponding to one of the diversity channels. When the channels are independently fading, it suffices to estimate each channel based only on the corresponding received signal. We concentrate on this case, and for simplicity drop the index designating the channel number.

In our case of a $T/2$ fractionally spaced feedforward filter, the inputs to the estimator are the two $T/2$ spaced samples of the received signal $\underline{v}(n) = [v(nT)\; v(nT - T/2)]^T$. The estimator uses a Kalman type algorithm based on the knowledge of the process and measurement equation

$$\mathbf{c}(n+1) = \mathbf{F}\mathbf{c}(n) + \boldsymbol{\chi}(n) \tag{51}$$

$$\underline{v}(n) = \mathbf{D}'(n)\mathbf{T}\mathbf{c}(n) + \underline{\nu}(n) \tag{52}$$

where the process noise is assumed to be white with covariance $\mathbf{Q}$, and $\underline{\nu}(n) = [\nu(nT)\,\nu(nT - T/2)]^T$ is the measurement noise of covariance $N_0\underline{G}$. However, the sequence $\{\underline{\nu}(n)\}$ is not white in general, since the receiver filter introduces correlation.[3] In order to deal with the colored noise, either an optimal filter of increased dimension can be designed, or a suboptimal filter of the original state dimension can be used, retaining comparable performance [10]. When the receiver filter is an integrator or a square root Nyquist filter with roll-off 1, the measurement noise vectors will be correlated only at the lag of one time instant. In such a case, the optimal filter has its complexity just slightly increased, and it achieves the same performance as if the noise were white. We are going to concentrate on the white noise case, keeping in mind that other choices of the receiver filter and adequate whitening procedures will result in somewhat increased, but comparable estimation error covariances.

The measurement equation involves a time dependent data vector

$$\mathbf{D}'(n) = [d(n-M_1)\mathbf{I}_2 \dots d(n)\mathbf{I}_2 \dots d(n+M_2)\mathbf{I}_2]. \tag{53}$$

Although it is given here in a noncausal form, in a real situation when the receiver operates in a decision-directed mode, there will exist a delay of $M_2$ symbol intervals in

[3] The $T$-spaced samples of the measurement noise are uncorrelated both in the case of rectangular receiver filter with cutoff frequency $1/T$, and when this filter has a square root Nyquist characteristic, but the samples spaced at an odd number of $T/2$ intervals are uncorrelated only in the first case.

the channel estimation process, and the channel estimates $\hat{\mathbf{h}}(m, n+1)$, computed upon forming the decision $\tilde{d}(n)$, will be obtained from the predictions

$$\hat{\mathbf{h}}(n-M_2+1+i|n-M_2) = \mathbf{T}\mathbf{F}^i\hat{\mathbf{c}}(n-M_2+1|n-M_2), \quad i \geq 0 \tag{54}$$

which are based on the currently available state estimate $\hat{\mathbf{c}}(n - M_2 + 1|n - M_2)$.

The state error covariance $\mathbf{\Delta} = Cov[\mathbf{c}(n) - \hat{\mathbf{c}}(n|n-1)]$ is defined through a discrete time Ricatti equation [10]

$$\begin{aligned} \mathbf{M}(n, \mathbf{D}(n)) &= \mathbf{D}(n)[\mathbf{D}'(n)\mathbf{T}\mathbf{\Delta}(n|n-1)\mathbf{T}'\mathbf{D}(n) \\ &\quad +N_0\underline{G}]^{-1}\mathbf{D}'(n) \\ \mathbf{\Delta}(n+1|n) &= \mathbf{F}[\mathbf{\Delta}(n|n-1) - \mathbf{\Delta}(n|n-1)\mathbf{T}' \\ &\quad \cdot\mathbf{M}(n, \mathbf{D}(n))\mathbf{T}\mathbf{\Delta}(n|n-1)]\mathbf{F}' + \mathbf{Q}. \end{aligned} \tag{55}$$

Due to the data-dependence, there is no steady-state solution to the Ricatti equation (55). On the other hand, for the case of random input data, one does not expect the error covariance matrix, or rather, the average probability of error, to depend heavily on the particular transmitted sequence. We may therefore resort to an approximation in order to obtain a quasi steady-state value of the error covariance matrix. By invoking once again the assumption about the independence of the steady-state error covariance to the data symbols, and averaging (55) with respect to the data sequence, we obtain the corresponding quasi stationary form of the Ricatti equation

$$\begin{aligned} \mathbf{M}(n) &= E\{\mathbf{D}(n)[\mathbf{D}'(n)\mathbf{T}\mathbf{\Delta}(n|n-1)\mathbf{T}'\mathbf{D}(n) \\ &\quad +N_0\underline{G}]^{-1}\mathbf{D}'(n)\} \\ \mathbf{\Delta}(n+1|n) &= \mathbf{F}[\mathbf{\Delta}(n|n-1) - \mathbf{\Delta}(n|n-1)\mathbf{T}' \\ &\quad \cdot\mathbf{M}(n)\mathbf{T}\mathbf{\Delta}(n|n-1)]\mathbf{F}' + \mathbf{Q}. \end{aligned} \tag{56}$$

When the simple model (49) is used, this equation can be rewritten directly in terms of the channel error covariance $\mathbf{E}(n|n-1) = Cov[\mathbf{h}(n) - \hat{\mathbf{h}}(n|n-1)]$ as

$$\begin{aligned} \mathbf{M}(n) &= E\{\mathbf{D}(n)[\mathbf{D}'\mathbf{E}(n|n-1)\mathbf{D}(n) \\ &\quad +N_0\underline{G}]^{-1}\mathbf{D}'(n)\} \\ \mathbf{E}(n+1|n) &= |f|^2[\mathbf{E}(n|n-1) - \mathbf{E}(n|n-1)\mathbf{M}(n) \\ &\quad \cdot\mathbf{E}(n|n-1)] + (1-|f|^2)\mathbf{H} \end{aligned} \tag{57}$$

where $\mathbf{H}$ is the sampled channel covariance, which can be measured at the receiver.

Finally, from the quasi steady-state value $\mathbf{\Delta}$, and the state covariance $\mathbf{C}$, the needed matrices $\mathbf{E}(m)$, $\mathbf{H}(m)$ are obtained. Assuming that the channel stays fixed for the duration of one symbol interval, the channel vectors $\mathbf{h}(m,n)$ can be represented as

$$\mathbf{h}(m,n) = \sum_{i=-M_2}^{M_1} \mathbf{J}_{i,i+m}\mathbf{h}(n+i) \tag{58}$$

where $\mathbf{J}_{i,i+m}$ is an appropriately defined transformation which assigns the two samples corresponding to the $(i+m)^{th}$ symbol interval of the vector $\mathbf{h}(n+i)$ to the $i^{th}$ symbol interval of the vector $\mathbf{h}(m,n)$, setting the rest to zero. The desired channel estimates are obtained as

$$\hat{\mathbf{h}}(m,n) = \sum_{i=-M_2}^{M_1} \mathbf{J}_{i,i+m}\hat{\mathbf{h}}(n+i|n-M_2-1). \tag{59}$$

From these expressions, we obtain the covariance matrices

$$\mathbf{H}(m) = \sum_{k,} \sum_{l=-M_2}^{M_1} \mathbf{J}_{k,k+m}\mathbf{T}\mathbf{C}_{k,l}\mathbf{T}'\mathbf{J}'_{l,l+m} \tag{60}$$

and

$$\mathbf{E}(m) = \sum_{k,} \sum_{l=-M_2}^{M_1} \mathbf{J}_{k,k+m}\mathbf{T}\mathbf{\Delta}_{k+M_2,l+M_2}\mathbf{T}'\mathbf{J}'_{l,l+m} \tag{61}$$

where

$$\mathbf{C}_{k,l} = \mathbf{F}^{k-l}\mathbf{C}, \quad k \geq l \tag{62}$$

and

$$\begin{aligned} \mathbf{\Delta}_{k,l} &= \mathbf{F}^k\mathbf{\Delta}\mathbf{F}'^l + \mathbf{F}^{k-l}\mathbf{Q}_{l,l}, \quad k \geq l, \\ \mathbf{Q}_{l,l} &= \sum_{i=0}^{l-1}\mathbf{F}^i\mathbf{Q}\mathbf{F}'^i, \quad l > 0, \quad \mathbf{Q}_{0,0} = \mathbf{0}. \end{aligned} \tag{63}$$

Using this approach with approximation of the error covariance matrix by its quasi steady-state value, several examples have been analyzed, and they are presented in the following section. For the chosen channel parameters, computer experiments were conducted to verify that the probability of error results obtained by averaging the solutions of the true Ricatti equation (55) showed excellent agreement with those obtained when the stationary solution of (57) was used for the computation of the error covariance matrices. Hence, either method can be used to obtain the desired data-independent error covariance; however, solving the modified Ricatti equation is computationally much simpler.

## VI. Numerical Examples

We illustrate the results obtained through several examples. A two-path Rayleigh fading model is used for each of the diversity channels. The paths are taken to be of equal power 1/2, and independently fading with a differential delay of half a symbol interval. The transmitter and the receiver filter pulses are assumed to be rectangular of duration $T$ and unit energy. A first-order Gauss-Markov model for the fading process as described by the equations (49) is used. The signal-to-noise ratio is defined as $SNR = E_b/N_0$, where $E_b = E_s/log_2\mathrm{M}$, M being the modulation level, and the signal energy is

$$E_s = \sum_m E\{|h(mT/2, nT)|^2\} = tr(\mathbf{H}). \tag{64}$$

Besides varying the $SNR$, we shall also be interested in the probability of error as a function of the channel parameter $f$. The parameter $f$ can be described as the value of the channel spaced-time correlation function at a spacing of one symbol interval. Since it is more common to use the Doppler spread of the channel to characterize its time-coherence properties, we first relate the parameter $f$ to the Doppler spread $B_d$. A constant transition matrix corresponds to an exponentially decaying channel time correlation function, in which case the desired relationship is

$$f = e^{-\omega_d T} \tag{65}$$

where $\omega_d = 2/(\Delta t)_c = \pi B_d$, $(\Delta t)_c$ is the coherence time of the channel, and $B_d$ the $3dB$ bandwidth of the related Doppler power spectrum.

Several examples are presented in Figs.2 through 5. In all of the examples it is assumed that the feedforward filter spans two symbol intervals, and that the feedback filter is as long as the causal ISI.

Fig.2 shows the probability of a bit error for BPSK signaling, as a function of the $SNR$, with normalized fading rate $\omega_d T$ as a parameter. The two sets of curves correspond to the single-channel and dual diversity reception. For values of $f$ very close to 1, there is little or no impact of the estimation errors on the overall performance; however, as $f$ decreases, i.e. fading gets faster, the degradation becomes significant and eventually limits the performance. The resulting error floor is higher for higher fading rates. The reason for the error probability saturation is that even as the noise vanishes, there remains a constant, nonzero level of the error covariance matrix, induced by the channel dynamics only. These estimation errors lead to an irreducible bit error rate regardless of the residual ISI. Residual ISI alone will also lead to performance saturation [1], but at a higher $SNR$. Errors in channel estimation cause the critical $SNR$ at which saturation is reached to decrease as a function of fading rate.

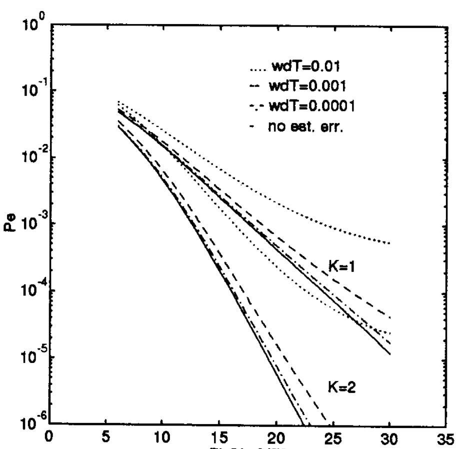


Fig. 2. Single-channel and dual diversity BPSK performance.

The single-channel BPSK probability of error is shown in Fig.3 as a function of fading rate for different $SNR$'s. The fading rate is represented through the quantity $(1-f)$, so that the limiting behavior as $\omega_d T \to \infty$ is concentrated in a single point. The 'no noise' curve represents the asymptotic, or irreducible error rate. For fading rates above $10^{-2}$, performance becomes severely limited on this channel. The degradation of the error probability from the corresponding value on a very slowly fading channel is highly dependent on the given $SNR$. At $SNR = 30\ dB$, an increase in fading rate from $\omega_d T = 10^{-4}$ to $\omega_d T = 10^{-2}$ causes the probability of error to increase about 30 times. However, for $SNR$'s of about 10 $dB$ or lower, the performance degradation due to channel estimation errors is relatively small at fading rates up to $10^{-2}$. At $SNR = 10\ dB$ and $\omega_d T = 10^{-2}$, the error probability is less than twice the value obtained in the absence of estimation errors.

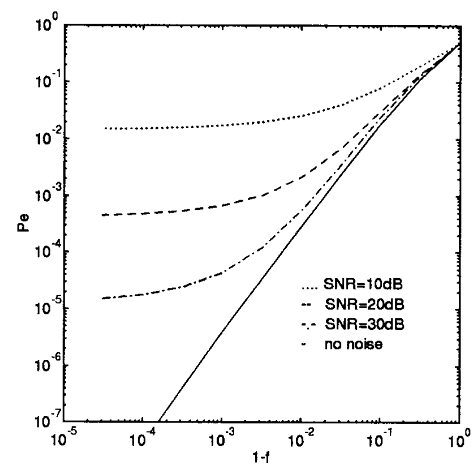


Fig. 3. Single-channel BPSK performance as a function of fading rate.

The second set of curves ($K=2$) in Fig.2 shows BPSK performance with dual diversity. While it is known that diversity helps to reduce the ISI [1], it helps less so to reduce the estimation error penalty. Since each independent diversity branch introduces new estimation errors, the estimation error penalty will increase with diversity order. Defining the BPSK estimation error penalty $E_2^{(k)}(\omega_d T, SNR)$ as the ratio of bit error probabilities at a given fading rate $\omega_d T$ and in the absence of estimation errors, the ratio of penalties with and without diversity, $E_{(2)}^{(2)}/E_2^{(1)}$, is plotted in Fig.4 versus $SNR$. At $SNR = 20\ dB$, and $\omega_d T = 10^{-3}$, the estimation error penalty is almost two times higher when diversity is employed. Nevertheless, its absolute value remains small, $E_2^{(2)}(10^{-3}, 20\ dB) = 3$. Similarly, as the number of propagation paths increases, the estimation error penalty will increase. Although in such a case the ISI penalty will also increase, better performance can still be achieved, which is explained by the fact that more implicit diversity is present in the longer multipath [1].

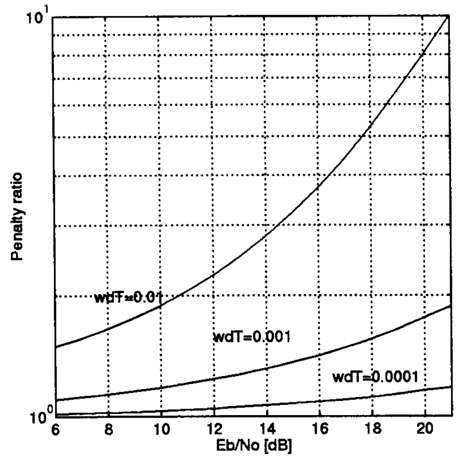


Fig. 4. BPSK estimation error penalty ratio for dual and no diversity.

Fig.5 shows the comparison between the results obtained for BPSK and QPSK signaling, in configurations with a single-channel and dual diversity. Similarly as for the nondispersive channels discussed in [4], the penalty caused by imperfectly estimating a multipath channel is higher for higher level modulation schemes. As this penalty increases with the order of diversity, so does the degradation $E_4^{(k)}/E_2^{(k)}$ between QPSK and BPSK performance.

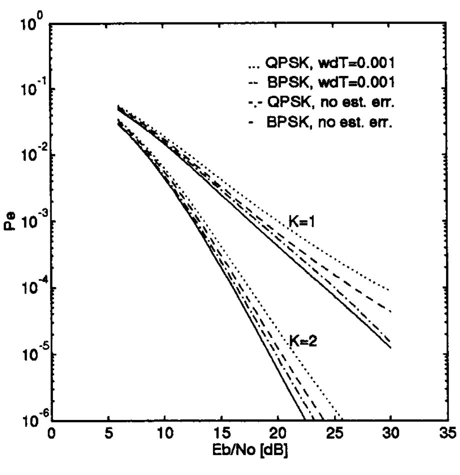


Fig. 5. BPSK and QPSK probabilities of bit error for single-channel and dual diversity reception.

The dominant component of the estimation error penalty lies in the error covariance $\mathbf{E}(0)$, which indicates that not only the DFE, but any other type of an equalizer, will suffer in a similar way from the incomplete knowledge of the channel.

The examples presented take into account only the channel estimation errors, assuming perfect knowledge of the channel fading model, which, of course, is generally not available. However, it has been shown (e.g., [6], [7]) that the Kalman filter is fairly robust to the model parameters mismatch, so that the additional degradation due to the model mismatch can be expected to be much smaller than that of the channel mismatch. Hence, the results presented can be regarded as the best-case performance of more realistic receivers.

## VII. Concluding Remarks

A coherent MPSK receiver with an adaptive multichannel DFE operating on a frequency selective, Rayleigh fading channel was considered. The expression for the probability of error, which takes into account the channel estimation errors, was presented. For the channel fading process described by a Gauss-Markov model, the model parameters, which describe fading dynamics, are reflected in the expression for the probability of error. The results presented show the penalty imposed by imperfect channel estimation, as well as the resulting irreducible error rates.

For a given fading channel, the normalized fading rate, which determines the estimation error penalty, will be lower at higher symbol rates, resulting in a smaller penalty due to the imperfect channel estimation. At the same time, the multipath will span more symbol intervals, resulting in a higher residual ISI penalty. This suggests a tradeoff in the determination of the symbol rate to be used for a given fading channel.

Relatively high losses which occur at high fading rates suggest that the commonly used performance measures for the DFE may be too loose in cases of rapidly fading channels. The degradation due to the channel mismatch then becomes significant, and has to be taken into account when estimating the achievable performance of a DFE.

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