Communication Theoretic Analysis of Underwater Ad-Hoc Networks in the Presence of Interference

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***Abstract*— We consider the performance of underwater acoustic ad-hoc networks in the presence of interference. We assume a uniform distribution of nodes over a finite area. The node-to-node channel is modeled using frequency dependent path loss and Ricean fading. We adopt a communication theoretic approach and study the interdependence of the sustainable number of hops through the network, end-to-end frame error probability, power and bandwidth allocation. The network operation is highly dependent on the node density with two distinct regions of limited performance: the coverage-limited region, where the number of nodes in the network is small, and the interference-limited region, where the number of nodes is large. We show that a desired level of connectivity can be achieved through a judicious selection of the operating frequency, power and bandwidth. Numerical examples are presented that illustrate the results of the analysis.**

I. Introduction

The design and analysis of underwater wireless (acoustic) communications systems has received an increased interest by both researchers and practitioners in the area [1]. Initial research efforts have focused on point-to-point communications giving rise to many applications that require an understanding of underwater networking principles. The study of underwater ad-hoc networks is therefore of paramount importance for the design of underwater monitoring sytems.

The performance of underwater acoustic ad-hoc networks was addressed in [2] under the idealized assumption that there is no interference in the network. In this paper, we extend the results of [2] to include the effect of interference. We focus on an underwater network of bottom mounted nodes, thus we consider a two-dimensional network model. Underwater acoustic transmissions are subject to high attenuation that depends both on the distance and the frequency of the signal. Hence, we assume multihop transmission based on nearest neighbor routing, as it offers more beneficial bandwidth and path loss conditions [3]. We adopt a communication theoretic approach [4] and investigate the network performance in the presence of interference from other nodes in the network. We focus on the interdependence between the sustainable number of hops in the network, as an indicator of network connectivity; end-to-end frame error probability, power and bandwidth allocation.

The paper is organized as follows. Section II describes the underwater acoustic propagation model.[1] Section III describes the ad-hoc network set-up and the communication theoretic analysis. Numerical examples illustrating the results of the analysis are presented in Section IV. We conclude in Section V.

II. Underwater Acoustic Propagation Model

A. Attenuation

Attenuation, or path loss, that occurs in an underwater acoustic channel over a distance d for a signal of frequency f, is given by

$$A(d,f) = A_0 d^{\kappa} a(f)^d \tag{1}$$

where A_0 is a unit-normalizing constant, $a(f)$ is the absorption coefficient and κ is the spreading factor. In the case of practical spreading, $\kappa = 1.5$. The absorption coefficient can be expressed empirically, using Thorp's formula which gives $a(f)$ in dB/km for f in kHz as [5]

$$10\log a(f) = \frac{0.11 f^2}{1+f^2} + \frac{44 f^2}{4100+f^2} + \frac{2.75 f^2}{10^4} + 0.003. \tag{2}$$

This formula is generally valid for frequencies above a few hundred Hz, which are typically used by acoustic modems.

B. Noise

The ambient noise in the ocean can be modeled using four sources: turbulence, shipping, waves and thermal noise. Most of the ambient noise sources can be described by Gaussian statistics and a continuous power spectral density. The following empirical formulae give the power spectral densities of the four noise components in dB re μ Pa per Hz as a function of frequency in kHz [5]

$$\begin{aligned}
10\log N_{\rm t}(f) &= 17 - 30\log f,\\
10\log N_{\rm s}(f) &= 40 + 20(s-0.5) + 26\log f\\
&\quad -60\log(f+0.03),\\
10\log N_{\rm w}(f) &= 50 + 7.5\sqrt{w} + 20\log f\\
&\quad -40\log(f+0.4),\\
10\log N_{\rm th}(f) &= -15 + 20\log f
\end{aligned} \tag{3}$$

where s is the shipping activity factor, $0 \le s \le 1$, and w is the wind speed in m/s. The overall power spectral density of the ambient noise is $N(f) = N_{\rm t}(f) + N_{\rm s}(f) + N_{\rm w}(f) + N_{\rm th}(f)$.

[1]Note that an acoustic signal propagates as a pressure wave whose level is commonly measured in dB relative to 1 μ Pa.

III. Ad-hoc Network Setup

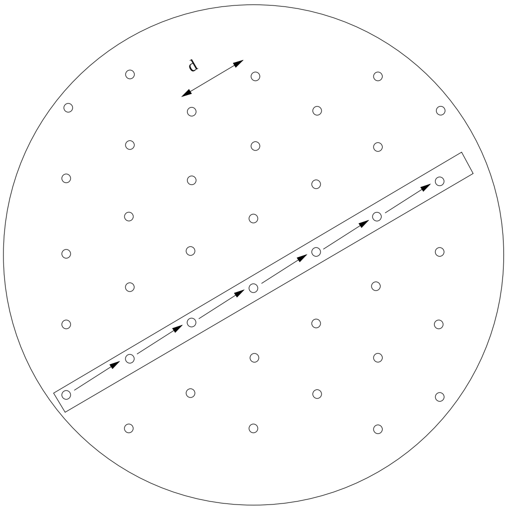

Fig. 1. Uniform network setup.

A. Network Topology

We consider a network of bottom mounted nodes, i.e., a two dimensional network that provides coverage over a certain area. Let the area of the network be a circle of radius r. We assume a uniform distribution of nodes in the network as depicted in Figure 1. Given the number of nodes in the network, N, and the area of the network, $\mathcal{A}$, the density of the network is

$$\rho_s = \frac{N}{\mathcal{A}}. \tag{4}$$

Given the uniform node distribution, the distance between the nodes is

$$d = \frac{c}{\sqrt{\rho_s}} \tag{5}$$

where c is a constant that depends on the node placement (grid pattern). Without loss of generality we assume that $c = 1$.

We assume multihop transmission based on nearest neighbor routing. This is an energy saving strategy, hence it may be attractive for networks with battery-powered nodes. As the longest multihop route in the network is along the diameter of the network, $D = 2\sqrt{\mathcal{A}/\pi}$, the maximum number of hops, $n_h^{\max}$ is

$$n_h^{\max} = \frac{D}{d} = \frac{2}{\sqrt{\pi}}\sqrt{N}. \tag{6}$$

Let the average number of hops for a multihop route be denoted as $\overline{n}_h$. Then, as long as the probability distribution of the number of hops is symmetric, that is, as long as very long and very short routes are much less likely than routes with an average number of hops, we have [4]

$$\overline{n}_h = \frac{n_h^{\max}}{2} = \sqrt{\frac{N}{\pi}}. \tag{7}$$

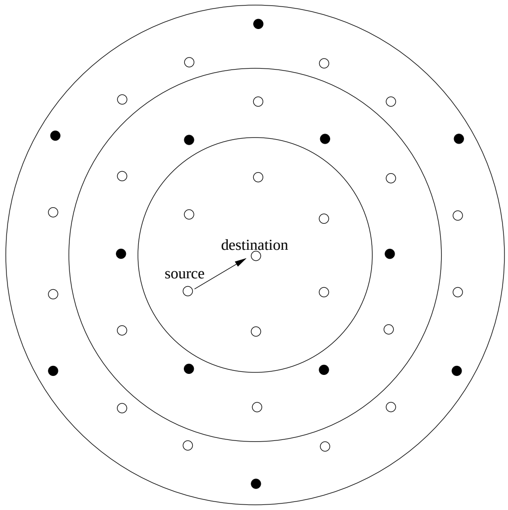

Fig. 2. Interfering nodes in the network.

B. Interference Model

In order to illustrate the effect of interference, we focus on a single transmission from a source node to a destination node, as depicted in Figure 2. We impose a protocol constraint: no nodes that are at the same distance from the destination node as the source node are allowed to transmit in the same time slot and in the same frequency band as the source during the source node's transmission.

The remaining nodes that may interfere with the source's transmission are organized in tiers. Assuming a hexagonal grid as an approximation of the network topology, there will be at most 12 interfering nodes in tier 1, 18 interfering nodes in tier 2, etc. A scenario where all nodes in the network transmit at the same time is unrealistic, as some nodes will be receiving while others transmit. Figure 2 depicts a scenario where a half of the nodes in tier 1 and a third of the nodes in tier 2 transmit at the same time and in the same frequency band as the source, thus creating interference. Assuming that all the nodes transmit with some constant power spectral density (p.s.d.) S, the combined interference from the nodes in the first and second tier is

$$I(f) \approx \frac{c_1 S}{A(2d,f)} + \frac{c_2 S}{A(3d,f)} \tag{8}$$

where $c_1 \leq 12$ and $c_2 \leq 18$ are constants indicating the number of interfering nodes in tiers 1 and 2, respectively. In the example presented in Figure 2, we have $c_1 = c_2 = 6$. As there are multiple interfering nodes in the network, we assume that the interference is Gaussian with p.s.d. $I(f)$.

Using the attenuation $A(d,f)$, the noise p.s.d. $N(f)$ and the interference p.s.d. $I(f)$, we can evaluate the signal to interference plus noise ratio (SINR) observed over a distance d, as shown in Figure 3. We observe that there is a preferred operating frequency, $f_o(d)$, which depends on the distance, d. The factor $[A(d,f)(N(f)+I(f))]^{-1}$ is maximized at this frequency. Figure 4 presents this preferred operating frequency as a function of the distance, given a transmit p.s.d. level of $S = 150$ dB re μ Pa per Hz for f in kHz.

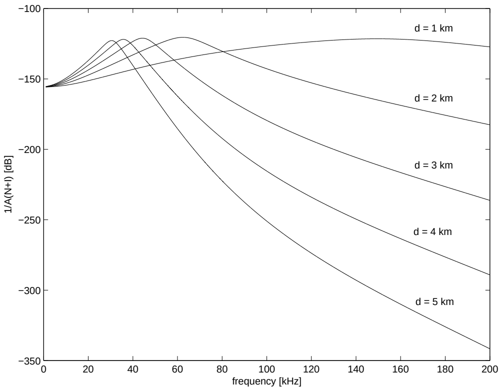

Fig. 3. The signal to noise plus interference ratio for various distances d. The transmit power spectral density is $S = 150$ dB re μ Pa per Hz. The spreading factor is $\kappa = 1.5$.

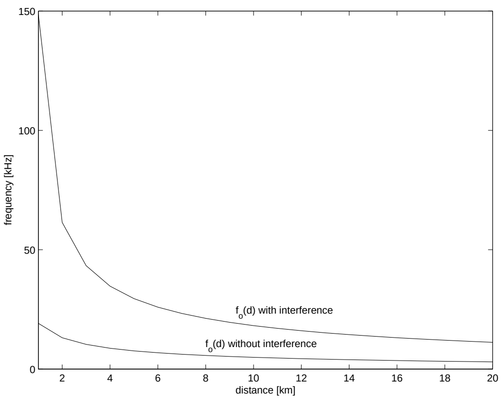

Fig. 4. Operating frequency $f_o(d)$. The transmit power spectral density is $S = 150$ dB re μ Pa per Hz. The spreading factor is $\kappa = 1.5$.

C. Multihop Transmission

We assume uncoded BPSK transmission with a simple demodulate-and-forward relaying strategy. The end-to-end frame error probability (FEP) for a multihop route with n_h hops is given by

$$p_{\rm route} = 1 - (1 - p_b)^{L n_h} \tag{9}$$

where p_b denotes the bit error probability of a single node-to-node link, and L denotes the frame size in bits.

We consider the quality-of-service for the network in terms of the maximum allowed end-to-end route FEP, i.e., we require that $p_{\rm route} \leq p_{\rm route}^{\rm max}$. Let the number of hops that can be sustained by the network, i.e., the number of hops that can satisfy the maximum end-to-end route FEP, be denoted by $n_{\rm sh}$. From Eq. (9), it follows that $n_{\rm sh}$ can be calculated as

$$n_{\rm sh} = \frac{1}{L}\frac{\log(1 - p_{\rm route}^{\rm max})}{\log(1 - p_b)} \approx \frac{1}{L}\frac{p_{\rm route}^{\rm max}}{p_b}. \tag{10}$$

While the analysis does not consider it explicitly, note that $n_{\rm sh}$ and $n_h^{\rm max}$ will in practice be chosen as nearest integers. Under the assumption of a Ricean fading model for the node-to-node channel [6], and assuming that perfect channel state information is available at the receiver, the bit error probability, p_b, can be upper bounded as [7]

$$p_b \leq \frac{1+\mathcal{K}}{1+\mathcal{K}+\gamma(d,f)} \exp\left(-\frac{\mathcal{K}\gamma(d,f)}{1+\mathcal{K}+\gamma(d,f)}\right) \tag{11}$$

where $\mathcal{K}$ denotes the Ricean fading factor and γ denotes the SINR. We assume that the attenuation, noise and interference are constant over the operational bandwidth B, so that the SINR can be calculated at the operating frequency $f_o(d)$ as

$$\gamma(d, f_o) = \frac{P}{A(d, f_o)(N(f_o) + I(f_o))B}. \tag{12}$$

The frequency-nonselective assumption is a suitable approximation for systems with narrow bandwidth. It can also be extended to wideband multi-carrier systems, such as OFDM. In that case, the operating frequency, $f_o(d)$, would describe the performance on one of the carriers. The performance on the other carriers would correspond to the frequency $f_o(d)$ shifted by multiples of subcarrier separation Δf.

IV. Numerical Results

We present numerical examples that examine the relationships between the sustainable number of hops, the end-to-end FEP, the signal power and the bandwidth. We assume Ricean fading for each node-to-node channel [6]. The Ricean fading factor is taken to be $\mathcal{K}$ = 10. We assume circular network of area $\mathcal{A} = 1000$ km^2. We assume the attenuation model given in Eq. (1) and neglect any fixed losses.[2] The frame size is L = 100 bits. The spreading factor is κ = 1.5, the shipping activity factor is s = 0.5, and we assume calm conditions, that is, the wind speed is w = 0 m/s.

Figure 5 presents the sustainable number of hops for an end-to-end FEP of 10^{-2}, bandwidth $B = 4$ kHz and transmit power $P = 140$ dB re μ Pa. The nodes are able to adjust their powers, so that the sustainable number of hops through the network never exceeds the maximum number of hops given in Eq. (6). The average number of hops given by Eq. (7) is also presented. We observe that when there are $N \lesssim 30$ nodes, the network cannot provide connectivity. This is due to the fact that with so few nodes in the network, the nodes are too far apart to guarantee the required end-to-end FEP for the available transmit power. Hence, the network is coverage-limited. Contrary to this situation, when the number of nodes in the network is $N \gtrsim 560$, we observe that the network can no longer sustain routes with the maximum number of hops. As we increase the number of nodes, while keeping the area of the network constant, the distance between the nodes decreases and the interference becomes stronger. Hence, the sustainable

[2]Inclusion of additional frequency independent losses, and an adjustment of the background noise level to suit a particular environement and provide the necessary SINR margins, will scale the results in absolute value, but will not alter the general behavior.

number of hops begins to decrease. We note however, that even with $N = 1000$ nodes in the network, the network can still maintain routes with an average number of hops. Nonetheless, the network is interference-limited. When the number of nodes in the network is between these values, $30 \lesssim N \lesssim 560$, the network can provide full connectivity and meet the target end-to-end FEP.

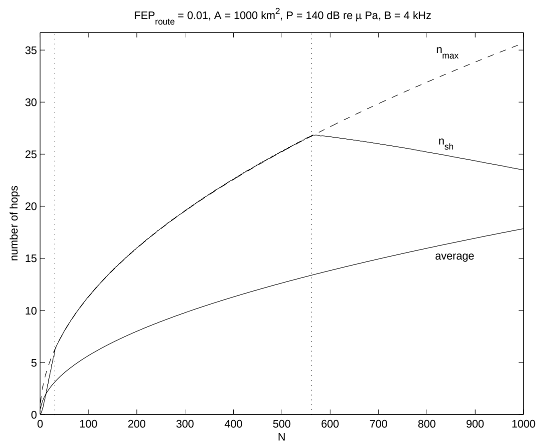

Fig. 5. Sustainable number of node-to-node hops for a uniform network with Ricean fading. The area is $\mathcal{A} = 1000$ km^2, the bandwidth is $B = 4$ kHz, the transmit power is $P = 140$ dB re μ Pa.

Figure 6 presents the sustainable number of hops for different values of the power. The end-to-end FEP is 10^{-3}. The bandwidth is $B = 4$ kHz. When the power is $P = 168$ dB re μ Pa the network provides full connectivity for all values of $N < 1000$. If the power is decreased to $P = 164$ or 160 dB re μ Pa, the network becomes interference-limited when the number of nodes in the network is $N \gtrsim 700$ and $N \gtrsim 400$, respectively. The corresponding operating frequency and the minimum transmit power needed to achieve full connectivity, i.e., $n_{\rm sh} = n_h^{\max}$, are illustrated in Figure 7.

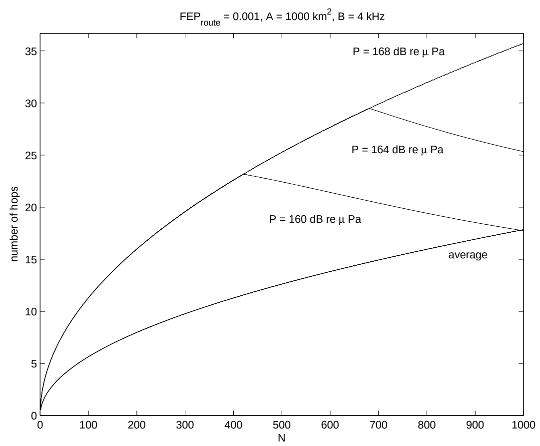

Fig. 6. Sustainable number of node-to-node hops for a uniform network with Ricean fading for various powers P. The area is $\mathcal{A} = 1000$ km^2, the bandwidth is $B = 4$ kHz.

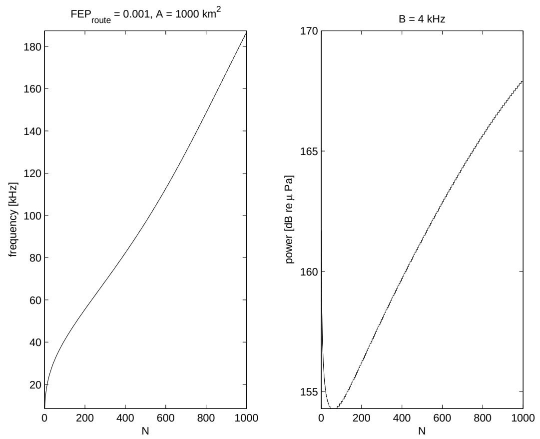

Fig. 7. The operating frequency and transmit power for a uniform network with Ricean fading. The area is $\mathcal{A} = 1000$ km^2, the bandwidth is $B = 4$ kHz.

Figure 8 depicts the sustainable number of hops for different values of the bandwidth. The end-to-end FEP is 10^{-3}. The power is $P = 164$ dB re μ Pa. We observe that the network behavior changes as we vary the bandwidth. For example, when the bandwidth is $B = 1$ kHz the network provides full connectivity for all values of $N < 1000$. When the bandwidth is $B = 2.5$ kHz, the network becomes interference-limited when the number of nodes is $N \gtrsim 830$. When the bandwidth is increased to $B = 4$ kHz, the network becomes interference-limited when the number of nodes exceeds $N \gtrsim 700$. Note that this behavior is not inherent to the channel, but is rather due to the fact that we have used the same signal power in all three cases. In other words, while the signal power remains the same, the noise power increases with the increased bandwidth causing an overall degradation in the system performance.

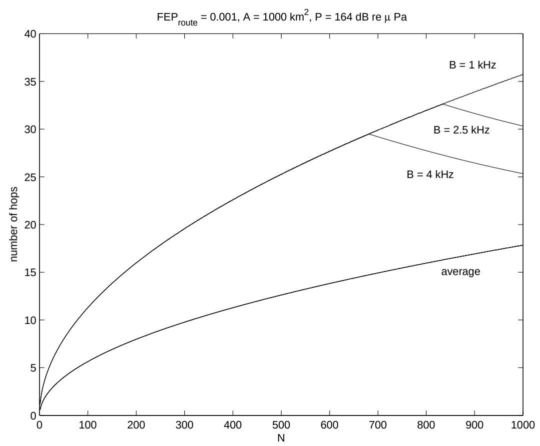

Fig. 8. Sustainable number of node-to-node hops for a uniform network with Ricean fading for various bandwidths. The area is $\mathcal{A} = 1000$ km^2, the transmit power is $P = 164$ dB re μ Pa.

Figure 9 presents the sustainable number of hops for various values of the end-to-end FEP and transmit power. The bandwidth is $B = 4$ kHz. We observe that for an end-to-end FEP of 10^{-3} and $P = 164$ dB re μ Pa, the network becomes interference-limited when the number of nodes is $N \gtrsim 700$. In this case the nodes have sufficient transmit power so that the network does not become coverage-limited when the number of nodes is small. When the end-to-end FEP is 10^{-2} and $P = 138$ dB re μ Pa, the network can provide full connectivity when the number of nodes is between $70 \lesssim N \lesssim 300$. Above $N \gtrsim 300$ nodes, the network becomes interference-limited. In contrast, when the number of nodes in the network is less than $N \lesssim 70$ nodes, the network is coverage-limited.

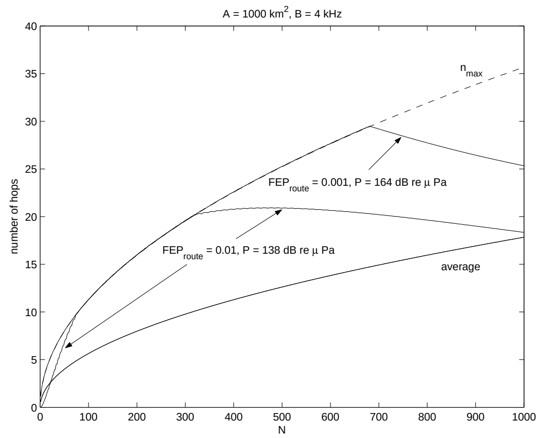

Fig. 9. Sustainable number of node-to-node hops for a uniform network with Ricean fading for various end-to-end frame error probabilities and transmit powers. The area is $\mathcal{A} = 1000$ km^2.

The sensitivity of the sustainable number of hops to the carrier frequency is addressed in Figure 10. The end-to-end FEP is 10^{-3}, the bandwidth is $B = 4$ kHz and the transmit power is $P = 164$ dB re μ Pa. We observe that when f_o is chosen as the operating frequency, the network becomes interference-limited when the number of nodes is $N \gtrsim 700$. At this frequency, the transmit power is sufficiently high that the network is not coverage-limited even for a small number of nodes. If the carrier frequency is $f_o \pm 5$ kHz, the network becomes interference-limited when the number of nodes is $N \gtrsim 650$ and coverage-limited when the number of nodes is $N \lesssim 80$. When the carrier frequency is $f_o \pm 8$ kHz, the changes in the operating regions of the network are more pronounced. We observe that when the number of nodes is $N \gtrsim 600$ nodes, the network becomes interference-limited. The network is also coverage-limited when the number of nodes in the network is $N \lesssim 250$. The network behaves in this way because a deviation from the preferred operating frequency f_o causes the SINR to decrease, as depicted in Figure 3, and consequently, the sustainable number of hops decreases as well.

V. Conclusions

We considered a communication theoretic analysis of underwater acoustic ad-hoc networks in the presence of interference. In particular, we studied the interdependence between the sustainable number of hops in the network, end-to-end FEP, power and bandwidth allocation. We found that the region

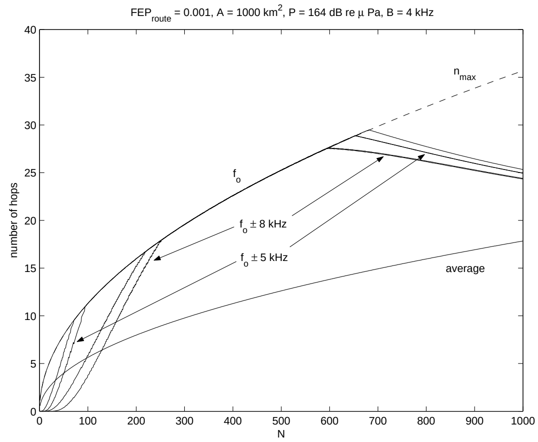

Fig. 10. Sustainable number of node-to-node hops for a uniform network with Ricean fading for various carrier frequencies: f_o, $f_o \pm 5$ kHz and $f_o \pm 8$ kHz. The area is $\mathcal{A} = 1000$ km^2, the bandwidth is $B = 4$ kHz, the transmit power is $P = 164$ dB re μ Pa.

of the network where it provides full connectivity may be limited from below by coverage and from above by interference. When the number of nodes in the network is small, such that the available power is not sufficient to provide connectivity over a (large) area, the network is coverage-limited. As the number of nodes in the network increases while the network area remains constant, the distance between the nodes decreases, and the network becomes interference-limited. Both the coverage-limited region and the interference-limited region can be controled through a proper choice of the operating frequency and the transmit power. Specifically, when the operating frequency is chosen to maximize the SINR, the range of supported node densities (those for which full connectivity can be established) is widest. In contrast, if the operating frequency deviates from this value, or if the power is reduced from the minimum needed to achieve full connectivity, the range of supported densities narrows. Numerical examples support these findings.

Acknowledgments

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