Estimation and Tracking of Time-Varying Channels in OFDM Systems

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***Abstract*— We propose a method for channel estimation in orthogonal frequency division multiplexing (OFDM) systems. Unlike the conventional sample-spaced and sub-sample-spaced methods, which target the *taps* of an equivalent discrete-delay channel response, this method targets the physical propagation *paths* in a continuous-delay domain. Numerical results quantify its benefits over the conventional least-squares, orthogonal matching pursuit, and mixed-norm methods. Assuming that channel is slowly varying from one OFDM block to another, we also develop an adaptive algorithm for channel tracking. Performance is illustrated through simulation.**

I. INTRODUCTION

Pilot-assisted channel estimation is used as a standard method to obtain the necessary channel state information (CSI) for reliable coherent communications. The conventional approaches to channel estimation, such as least-square (LS) estimators, are based on the sample-spaced channel models. Although these methods have low complexity, their performance suffers when the channel is not sample-spaced, as is the case in most practical situations. In [1], a channel estimator for OFDM systems has been proposed based on the singular-value decomposition (SVD) or frequency-domain filtering. While this estimator is optimal in the sense of mean-squared error (MSE), it requires a-priori information about the channel statistics, which is not usually available in practice.

With the advent of sparse estimation, focus has come to the sparse nature of physical multipath channels, as it can be leveraged to improve channel estimation to either work with fewer pilot symbols or to achieve better noise suppression. In [2], a channel estimator based on the greedy orthogonal matching pursuit (OMP) using dictionaries with finer delay resolution (sub-sample spacing) has been addressed. This method can approximately reflect the fact that the physical path delays have a continuum of values. However, the improvement in the performance of channel estimation which results from using finer, over-complete dictionaries comes at increased computational complexity, and further studies revealed a strongly diminishing return for finer dictionaries [3].

The method proposed in this paper focuses on explicit estimation of delays and complex amplitudes of the channel paths. Not only does it operate in a continuous estimation space, but it also eliminates the need to know the statistics of the channel, which is crucial for the SVD-based channel estimation. This method also has the capability to track the time-variation of the channels, such as underwater acoustic channels which experience the motion-induced Doppler shift.

We focus on a physical, path-based channel model, amenable to explicit channel estimation, where the channel is parametrized by a number of distinct paths, each characterized by a triplet of delay, complex amplitude and Doppler factor. We also relate the problem of channel estimation to that of direction finding in the array processing literature. Considering the fact that many practical wireless channels do not change much from one OFDM block to the next, we introduce a specific tracking algorithm to jointly track the path delays and the complex amplitudes. We investigate the performance of the proposed channel estimator through simulation.

The rest of this paper is organized as follows. In Sec. II, we introduce the system and channel models. Sec. III briefly explores least-squares (LS), matching pursuit (MP) and its orthogonal variant, and basis pursuit (BP) algorithms which are based on the discrete-delay model. In Secs. IV and V we detail our channel estimation and tracking method. Sec. VI contains the results of numerical simulations which quantify the effect of delay resolution and Doppler factor on the performance of the algorithms. We conclude in Sec. VII.

II. SYSTEM AND CHANNEL MODEL

We consider an OFDM system with K carriers and a total bandwidth B. We denote by f_0 and $\Delta f = B/K$ the frequency of the first carrier and the carrier separation, respectively. We assume that the use of a cyclic prefix of length L preserves the orthogonality of the carriers and eliminates inter-block interference between consecutive OFDM blocks. Channel is assumed to be slowly fading and constant during one OFDM block. Under these assumptions, the signal received on the k-th carrier can be described as

$$y_k = d_k H_k + z_k \tag{1}$$

where d_k is the transmitted data symbol, H_k is the channel frequency response at the frequency of the k-th carrier, and z_k is the additive Gaussian random variables with zero mean and variance σ_z^2. In matrix notation, we describe the OFDM system as

$$\mathbf{y} = \mathbf{D}\mathbf{H} + \mathbf{z} \tag{2}$$

where $\mathbf{y}$ is the received signal vector, $\mathbf{D}$ is a diagonal matrix containing the transmitted data symbols, $\mathbf{H}$ is the channel frequency response vector, and $\mathbf{z}$ is the vector of independent identically distributed (i.i.d) AWGN elements. The noise is assumed to be uncorrelated with the the channel $\mathbf{H}$.

We model the (actual, physical) propagation channel as

$$H(f) = \sum_p h_p e^{-j2\pi f_k \tau_p} \tag{3}$$

where h_p, τ_p represent the path gains and delays, respectively. At the carrier frequencies $f_k = f_0 + k\Delta f$, we have

$$H_k = \sum_p \underbrace{h_p e^{-j2\pi f_0 \tau_p}}_{c_p} e^{-j2\pi k \Delta f \tau_p}, \; k = 0, \dots K-1 \tag{4}$$

The delays τ_p have a continuum of values, i.e. they are not constrained to discrete, sample-spaced values.

A discrete-delay model with delay spacing $\Delta\tau$ is given by

$$H_k = \sum_l b_l e^{-j2\pi k \Delta f l \Delta\tau}, k = 0, \dots K-1 \tag{5}$$

III. CHANNEL ESTIMATION BASED ON THE DISCRETE-DELAY MODEL

Conventionally, channel estimation is based on the discrete-delay model. When $\Delta\tau = T/K$, with $T = 1/\Delta f$ we have the usual DFT relationship,

$$\mathbf{H} = \mathbf{F}_K \mathbf{b} \tag{6}$$

where $\mathbf{F}_K$ is the $K \times K$ Fourier matrix. If the multipath spread T_{mp} is within LT/K, then we can also write

$$\mathbf{H} = \mathbf{F}_{KL} \mathbf{b}_L \tag{7}$$

where only the first L columns of the Fourier matrix $\mathbf{F}_K$ and the first L elements of the model vector $\mathbf{b}$ are kept.

Assuming (without the loss of generality) that all K data symbols are available for channel estimation (e.g. correct symbol decisions, or all-pilots in an initial block), the input to the channel estimator is

$$\mathbf{x} = \mathbf{H} + \mathbf{z} \tag{8}$$

where $\mathbf{z}$ is the vector of i.i.d. AWGN elements.

The conventional LS estimate is then given by

$$\hat{\mathbf{b}}_L = \frac{1}{K} \mathbf{F}_{KL}^H \mathbf{x} \tag{9}$$

$$\hat{\mathbf{H}} = \mathbf{F}_{KL} \hat{\mathbf{b}}_L \tag{10}$$

In many practical applications, including underwater acoustic communications, the multipath channel can be considered as sparse, i.e. the number of significant paths is small even when the channel delay spread is long. Based on this assumption, the greedy matching pursuit (MP) and the basis pursuit (BP) algorithms can be used to estimate the channel response. The MP algorithm identifies the dominant channel taps sequentially. At each iteration it selects one column of the over-complete dictionary $\mathbf{F}_{KL}$ that correlates best with the approximation residual from the previous iteration and recomputes the coefficients by solving a constraint least-squares problem to optimally fit the observations[4]. In contrast, BP uses the l_1-norm to regularize the problem,

$$\min_{\mathbf{b}_L} \|\mathbf{x} - \mathbf{F}_{KL}\mathbf{b}_L\|_2^2 + \lambda \|\mathbf{b}_L\|_1 \tag{11}$$

where λ is tuned according to the variance of the noise. For the MP implementation we terminate the algorithm if the maximum correlation of the residual fitting error with the columns of the dictionary drops below a threshold based on the noise variance.

Alternatively, the "super-resolution" models and the attendant sparse estimators are based on using $\Delta\tau = T/KI$, where $I > 1$ accounts for an increased resolution in delay.

IV. CHANNEL ESTIMATION BASED ON THE PHYSICAL MODEL

The physical model is based on the expression (4), which can be written as

$$\mathbf{H} = \sum_p c_p \mathbf{s}(\phi_p), \quad \phi_p = 2\pi\Delta f \tau_p \tag{12}$$

where

$$\mathbf{s}(\phi) = \begin{bmatrix} 1 \\ e^{-i\phi} \\ \vdots \\ e^{-(K-1)\phi} \end{bmatrix} \tag{13}$$

represents a steering vector corresponding to an angle ϕ.

Consider now the following operation performed on the noisy channel observation $\mathbf{x}$:

$$r(\phi) = \frac{1}{K}\mathbf{s}^H(\phi)\mathbf{x} \tag{14}$$

This newly formed signal can be expressed as

$$r(\phi) = \sum_p c_p g(\phi - \phi_p) + n(\phi) \tag{15}$$

where

$$g(\phi) = \frac{1}{K}\sum_{k=0}^{K-1} e^{ik\phi} \tag{16}$$

is the signature waveform and $n(\phi)$ is Gassian noise with variance $\frac{1}{K}\sigma_z^2$. The signature waveform (magnitude) is depicted in Fig. 1

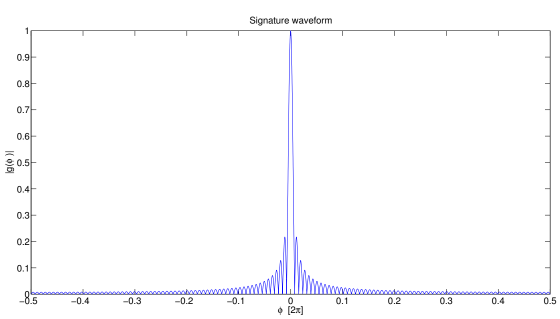

Fig. 1. Signature waveform.

The fact that the signature waveform is *known* can be exploited to estimate the channel parameters explicitly. When we say explicitly, we mean that we are targeting directly both the path gains c_p and the path delays τ_p (or more precisely the angles ϕ_p), unlike in the conventional estimation where the delay axis is discretized to avoid the non-linear problem of delay estimation.

Joint estimation of the parameters c_p and ϕ_p can be performed as follows. We start by setting

$$r_0(\phi) = r(\phi) \tag{17}$$

and evaluate this function for a pre-set range of angles ϕ with an arbitrary resolution $\Delta\phi$ in the angle domain. The range can be determined in accordance with the multipath spread T_{mp}. An iterative procedure now follows over the path indices $p = 0, 1, \ldots$. In the p-th iteration, we estimate the path angle as

$$\hat{\phi}_p = \arg\max_{\phi} |r_p(\phi)| \tag{18}$$

and the path coefficient as

$$\hat{c}_p = r_p(\phi) \tag{19}$$

We then subtract this path's contribution from the current signal, so as to form the signal for the next iteration

$$r_{p+1}(\phi) = r_p(\phi) - \hat{c}_p g(\phi - \hat{\phi}_p) \tag{20}$$

The procedure ends according to a pre-defined criterion

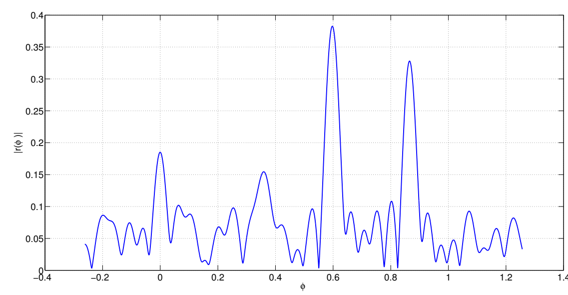

Fig. 2. Input signal for channel estimation.

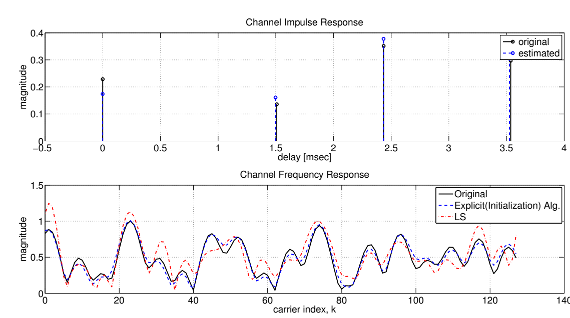

Fig. 3. Channel and its estimate.

such as an a-priori set number of paths, or when the power in the residual reaches a certain threshold or stops to change significantly.

Figs. 2 and 3 illustrate the algorithm performance on one OFDM block with the SNR of 5 dB. This algorithm is used for determining the initial values of the path delays.

An extension to the above algorithm can also be applied to improve the quality of the estimates $\hat{c}_p$. Once the algorithm has been executed, the path coefficients $\hat{c}_p$ generated in the process are discarded, but the angles $\hat{\phi}_p$ are kept. The angles are used to form the matrix

$$\hat{\mathbf{S}} = [\mathbf{s}(\hat{\phi}_0)\ldots\mathbf{s}(\hat{\phi}_{P-1})] \tag{21}$$

This matrix represents an estimate of a true matrix $\mathbf{S}$ which relates the vector $\mathbf{H}$ with the vector of path coefficients $\mathbf{c} = [c_0 \ldots c_{P-1}]^T$ as $\mathbf{H} = \mathbf{Sc}$, i.e. it defines the observed signal as

$$\mathbf{x} = \mathbf{Sc} + \mathbf{z} \tag{22}$$

The corresponding LS estimate is $\hat{\mathbf{c}} = [\mathbf{S}^H\mathbf{S}]^{-1}\mathbf{S}^H\mathbf{x}$. For lack of true $\mathbf{S}$, assuming that the angle estimates are accurate, we replace $\mathbf{S}$ by $\hat{\mathbf{S}}$. The channel coefficients are thus estimated as

$$\hat{\mathbf{c}} = [\hat{\mathbf{S}}^H\hat{\mathbf{S}}]^{-1}\hat{\mathbf{S}}^H\mathbf{x} \tag{23}$$

Unlike with the estimates (19), which are obtained sequentially (one after another), these estimates are obtained jointly, and hence offer a potential improvement. The algorithm is summarized in Table I.

TABLE I

PATH IDENTIFICATION ALGORITHM

```
data : r , number of paths (P)
initialization : r_0 = r
while p ≤ P do
    φ̂_p = argmax_φ |r_p|
    ĉ_p = r(φ̂_p)
    r_{p+1} = r_p − ĉ_p g(φ − φ̂_p)
end while
Ŝ = [s(φ̂_0)...s(φ̂_{P−1})], ĉ = [Ŝ^H Ŝ]^{−1} Ŝ^H x
```

V. TIME-VARYING CHANNEL AND TRACKING

A time-varying channel can be described by a set of time-varying path gains $h_p(n)$ and delays $\tau_p(n)$, where n has the notion of one OFDM block on a slowly-varying channel (no inter-carrier interference, just block-to-block variation).

Time-evolution of the path gains can be modeled as

$$h_p(n) = \rho_p h_p(n-1) + \chi_p(n) \tag{24}$$

where the path gain variance is $E\{|h_p|^2\} = \sigma_{h_p}^2$, and $\chi_p(n) \sim \mathscr{CN}(0, (1-\rho_p^2)\sigma_{h_p}^2)$ is the process noise which is uncorrelated with $h_p(n-1)$ as well as across p. The one-step correlation coefficient ρ_p is related the Doppler spread B_p of the p-th path via $\rho_p = e^{-\pi B_p(T+T_g)}$. Such a model can be used for both radio and acoustic channels [5].

Time-evolution of the path delays is modeled as

$$\tau_p(n) = \tau_p(n-1) - a_p(n)\cdot T' \tag{25}$$

where $T' = T + T_g$ is the total block duration (data plus guard interval), and $a_p(n)$ is the Doppler factor that captures motion-induced time scaling on the p-th path (residual scaling after front-end resampling). Doppler scaling factor can be treated as deterministic or random, time-invariant or time-varying, depending on the particular propagation circumstances and time intervals of observation. To generate a time-series of the channel parameters $h_p(n), \tau_p(n)$, we start with their nominal values h_p, τ_p obtained form the system geometry.

Time-varying path gains and delays $h_p(n), \tau_p(n)$ give rise to the time-varying path coefficients and angles $c_p(n), \phi_p(n)$, as well as time-varying equivalent discrete-delay response $\mathbf{b}(n)$. In each OFDM block we thus have a different channel response that has to be estimated. Channel estimation can be performed in each block from scratch (using pilots). However, there may be advantages to using specific tracking algorithms that exploit the fact that many practical channels do not change much from one block to the next. The adaptive RLS-LASSO algorithm [6] is an example of an algorithm that can be applied to track the discrete-delay response $\hat{\mathbf{b}}(n)$. Here, we describe an algorithm that targets the physical model parameters. Specifically, this algorithm performs joint tracking of the path coefficients $\hat{c}_p(n)$ and angles $\hat{\phi}_p(n)$.

Tracking is initialized by the original path identification procedure, i.e. by the estimates $\hat{c}_p(0), \hat{\phi}_p(0)$, which are assumed to be reasonably accurate. During the following blocks, updates $\hat{c}_p(n+1), \hat{\tau}_p(n+1)$ are computed from the existing values $\hat{c}_p(n), \tau_p(n)$. Tracking can proceed over a frame of N blocks, i.e. for $n = 0, 1, \ldots N-1$. At that point, the algorithm can be re-initialized.

To develop the tracking algorithm, we start by approximating the true parameters are time-invariant (on the grounds that they are slowly varying). Assuming that we have the estimates $\hat{c}_p$ and $\hat{\phi}_p$, we form

$$\hat{r}(\phi) = \sum_p \hat{c}_p g(\phi - \hat{\phi}_p) \tag{26}$$

The underlying error is

$$e(\phi) = r(\phi) - \hat{r}(\phi) = r(\phi) - \sum_p \hat{c}_p g(\phi - \hat{\phi}_p) \quad (27)$$

Note that $r(\phi)$ is known, so the error can be be computed. The corresponding squared error, averaged over the observation range ϕ_{obs} is

$$E = \int_{\phi_{obs}} |e^2(\phi)| d\phi \quad (28)$$

The relevant partial derivatives yield the gradients

$$\begin{aligned} \frac{\partial E}{\partial \hat{c}_p} &= \int_{\phi_{obs}} \frac{\partial e(\phi)}{\partial \hat{c}_p} e^*(\phi) d\phi \\ &= -\int_{\phi_{obs}} g(\phi - \hat{\phi}_p) e^*(\phi) d\phi \end{aligned} \quad (29)$$

and

$$\begin{aligned} \frac{\partial E}{\partial \hat{\phi}_p} &= 2Re \int_{\phi_{obs}} \frac{\partial e(\phi)}{\partial \hat{\phi}_p} e^*(\phi) d\phi \\ &= 2Re\ \hat{c}_p \int_{\phi_{obs}} \dot{g}(\phi - \hat{\phi}_p) e^*(\phi) d\phi \end{aligned} \quad (30)$$

where

$$\dot{g}(\phi) = \frac{dg(\phi)}{d\phi} = \frac{1}{K} j \sum_{k=0}^{K-1} k e^{jk\phi} \quad (31)$$

The above gradients exhibit nonlinear dependence on the angle (delay) estimates, preventing us from obtaining a closed-form solution. A stochastic gradient algorithm can be employed instead to approach the solution in a recursive manner. Cast into the time-adaptive framework, the recursion yields the updates

$$\begin{aligned} \hat{c}_p(n+1) &= \hat{c}_p(n) + \mu \int_{\phi_{obs}} g(\phi - \hat{\phi}_p(n)) e^*(\phi, n) d\phi \\ \hat{\phi}_p(n+1) &= \hat{\phi}_p(n) - \\ & \underbrace{\nu Re\ \hat{c}_p(n) \int_{\phi_{obs}} \dot{g}(\phi - \hat{\phi}_p(n)) e^*(\phi, n) d\phi}_{\varepsilon_p(n)} \end{aligned} \quad (32)$$

where μ and ν are the a-priori set (positive) tracking constants, and $e(\phi, n)$ is the error resulting from the estimate $\hat{r}(\phi, n)$ formed in the n-th iteration.

While this type of update is the simplest one, other types can be considered as well. For instance, the LMS-like coefficient update can be replaced by an RLS-like one (this would effectively correspond to replacing the optimization criterion based on instantaneous squared error by one based on exponentially weighted sum of past errors). To improve the angle (delay) tracking, a filtered error gradient can be used:

$$\hat{\phi}_p(n+1) = \hat{\phi}_p(n) - \nu_1 \varepsilon_p(n) - \nu_2 \sum_{m=1}^{n} \varepsilon_p(n-m) \quad (33)$$

where ν_1, ν_2 are now two delay tracking constants. Other filtering strategies are also possible, e.g. exponential weighting of the past values of $\varepsilon_p(n)$.

In a digital implementation, the integrals $\int_{\phi_{obs}} f(\phi) d\phi$ will be replaced by sums $\sum_{i \in \mathscr{I}_{\phi_{obs}}} f(i\Delta\phi)\Delta\phi$, where $\Delta\phi$ is the desired resolution, and $\mathscr{I}_{\phi_{obs}}$ the set of indices corresponding to ϕ_{obs}. Using shorthand notation $r(i\Delta\phi, n) = r_i(n)$, $e(i\Delta\phi, n) = e_i(n)$, a full tracking algorithm is specified by the following steps carried out for $n = 0, \ldots N-1$:

$$\begin{aligned} e_i(n) &= r_i(n) - \sum_p \hat{c}_p(n) g(i\Delta\phi - \hat{\phi}_p(n)),\ \forall i \in \mathscr{I}_{\phi_{obs}} \\ \hat{c}_p(n+1) &= \hat{c}_p(n) + \mu \sum_{i \in \mathscr{I}_{\phi_{obs}}} g(i\Delta\phi - \hat{\phi}_p(n)) e_i^*(n) \\ \hat{\phi}_p(n+1) &= \hat{\phi}_p(n) - \\ & \underbrace{\nu Re\{\ \hat{c}_p(n) \sum_{i \in \mathscr{I}_{\phi_{obs}}} \dot{g}(i\Delta\phi - \hat{\phi}_p(n)) e_i^*(n)\}}_{\varepsilon_p(n)} \end{aligned} \quad (34)$$

where μ and ν are the tracking constants (appropriately scaled). Extension to a second-order delay loop is obvious. Table II summarizes the tracking algorithm in a vector format.

TABLE II
PATH TRACKING ALGORITHM

1. Initialization : tracking is initialized using channel estimation procedure described in Table I, which yields the estimates $\hat{\mathbf{c}}(0), \underline{\hat{\phi}}(0)$.
2. Tracking: $n = 0, \cdots, N-1$

$$\mathbf{r}(n) = \begin{bmatrix} \mathbf{s}^H(-I_1\Delta\phi) \\ \vdots \\ \mathbf{s}^H(I_2\Delta\phi) \end{bmatrix} \mathbf{y}(n)$$

$[\mathbf{G}(n)]_{i,p} = g(i\Delta\phi - \hat{\phi}_p(n)), i = -I_1, \ldots I_2, p = 0, \ldots \hat{P}-1$
$[\dot{\mathbf{G}}(n)]_{i,p} = \dot{g}(i\Delta\phi - \hat{\phi}_p(n)), i = -I_1, \ldots I_2, p = 0, \ldots \hat{P}-1$
$\mathbf{e}(n) = \mathbf{r}(n) - \mathbf{G}(n)\hat{\mathbf{c}}(n)$
$\hat{\mathbf{c}}(n+1) = \hat{\mathbf{c}}(n) + \mu \mathbf{G}^H(n)\mathbf{e}(n)$
$\underline{\hat{\phi}}(n+1) = \underline{\hat{\phi}}(n) - \nu Re\{\text{Diag}[\hat{\mathbf{c}}(n)]\dot{\mathbf{G}}^T(n)\mathbf{e}^*(n)\}$

VI. NUMERICAL RESULTS

To assess the system performance, we focus on an example of an OFDM system operating in a bandwidth of 5 kHz, divided into 128 carriers with a total symbol period of 30.7 ms, of which 5.1 ms is devoted to the cyclic prefix. We use 32 equally-spaced pilots. The modulation on each carrier is QPSK. Randomly generated channels consist of 4 paths whose delays are uniformly chosen from $[0,T_g]$, and whose gains are complex-valued Gaussian variables with zero mean and variance $[1,0.81,0.64,0.49]$.

Fig. 4 shows the normalized mean-squared error (NMSE) versus SNR for the proposed estimator ("path identification" algorithm), LS, OMP (with oversampling factor I = 2,3,4), and BP (I = 2) estimators. The NMSE is defined as

$$NMSE = \frac{E\{\|\mathbf{H}-\hat{\mathbf{H}}\|^2\}}{E\{\|\mathbf{H}\|^2\}} \tag{35}$$

The results shown in Fig.4 represent an average over 3000 channel realizations.

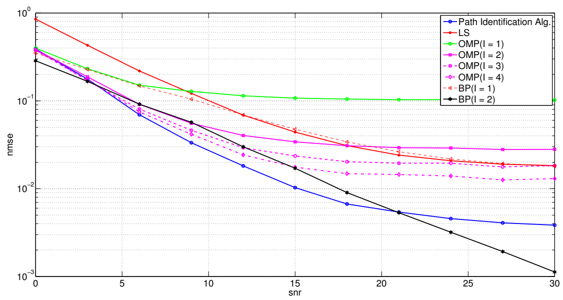

Fig. 4. Normalized mean-squared error for different estimators.

The OMP algorithm is restricted to operate with 4 taps, while the BP algorithm yields more than 4 significant taps (between 15 and 30 at the SNR of 30 dB). The explicit algorithm operates over the range of angles ϕ between $-\frac{\pi}{12}$ and $\frac{2\pi T_g}{T}$. Unlike the OMP and BP algorithms, whose performance saturates with a further increase in resolution (and possibly breaks down as well) the explicit algorithm can tolerate an arbitrary resolution in the ϕ domain. The results of Fig.4 indicate that path-based estimation outperforms all the other methods except the BP (I=2) at SNR>20 dB. Note, however, that there is a drastic difference in computational complexity of these two methods.

In Figs.5 and 6 we investigate the impact of Doppler scaling. To take the Doppler effect into account, we generate different OFDM frames each of which contains 32 OFDM blocks. As before, we use 3000 independent channel realizations for the first block of a frame, while subsequent blocks follow the models (24) and (25) for the path delay and gain variation. The results represent the average over all blocks and frames.

Fig.5 illustrates the case of Doppler scaling at $a_p = 10^{-4}$, while Fig.6 corresponds to $a_p = 10^{-3}$. In both cases, and especially at the higher Doppler factor, we note that explicit, path-based channel estimation outperforms the conventional methods. This phenomenon is explained by the failure of conventional methods to cope with path slipping. Namely, as the time evolves over one frame, Doppler causes the paths to move, eventually taking one (or more) outside of the algorithm's field of vision. At that point, observability is lost, and any algorithm that is based on a finite, fixed window of tap delays will fail. In contrast, explicit path identification is not constrained to a finite window of delay angles ϕ. In fact, it can entertain an arbitrarily large window of observation $\mathscr{I}_{\phi_{obs}}$, thus avoiding the loss of observability, and gaining performance over the conventional algorithms.

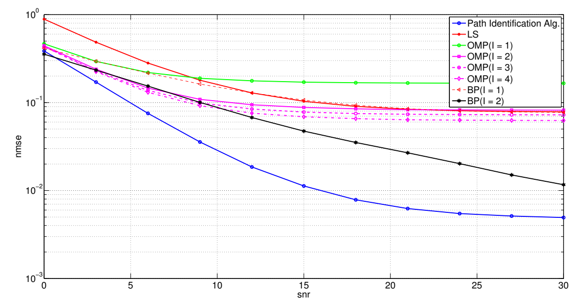

Fig. 5. NMSE vs. SNR, $a_p = 10^{-4}$, $p = 0,1,2,3$.

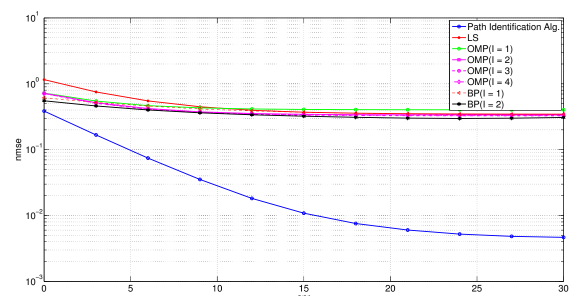

Fig. 6. NMSE vs. SNR, $a_p = 10^{-3}$, $p = 0,1,2,3$.

Finally, we address the performance of tracking in Fig. 7. This figure illustrates a set of 4 paths, showing the magnitude, phase and delay for each path. Plotted on top of the true values are the estimated values obtained by the tracking algorithm (33).

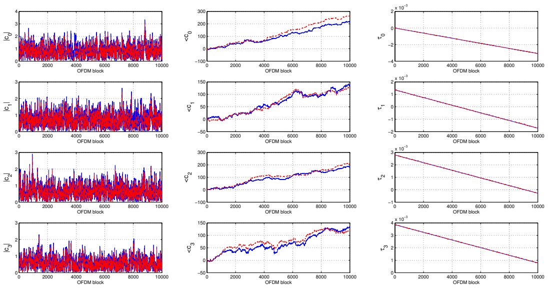

Fig. 7. Tracking algorithm: magnitude (left), phase (middle) and delay (right) of the channel paths. True and estimated values are shown in solid and dashed line. $a_p = 10^{-5}$, $p = 0,1,2,3$; $\mu = 0.02, \nu = 0.0000002$.

VII. CONCLUSION

We addressed the issue of channel estimation in OFDM systems, targeting explicitly the actual, physical propagation paths, rather than the equivalent taps of a discrete-delay impulse response model. The basic difference between the two approaches is that the former allows the path delays to have a continuum of values, while the latter restricts the taps to a pre-defined quantization grid. In pursuing the physical model, our goal was to reduce the total number of channel coefficients (there are fewer paths than taps), and thus improve the system performance in the presence of noise.

By employing a transformation of the signal received on a (large) number of OFDM carriers, the channel estimation problem was cast into a framework where path delays and gains are to be identified in a manner analogous to identifying angles of arrival in a typical beamforming (array processing) framework. A two-step procedure was then outlined, where the first step provides initial estimates of the path delays and gains, while the second step focuses on subsequent tracking.

Numerical results indicate superiority of the proposed method over the standard sparse estimation techniques. The major benefit of explicit tracking stems from the fact that its resolution and coverage in the delay domain can be increased arbitrarily, without a penalty on performance. This feature is of particular importance on channel with severe Doppler (e.g. acoustic channels) where path delays are prone to slipping.

Future research will focus on applications to real data, assessing the performance of the entire communication system, i.e. the corresponding bit error rate analysis, as well as on the extensions of channel tracking for situations with time-varying Doppler scaling.

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