Peak-to-Average Power Ratio (PAR) Reduction for Acoustic OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is an appealing modulation scheme for high-rate underwater acoustic communications which are challenged by multipath propagation. However, it has a drawback in the large peak-to-average power ratio (PAR). Techniques for PAR reduction have been extensively studied for radio communication systems. While these techniques are applicable to acoustic systems, we take a different approach that aims to capitalize on the fundamental differences between the acoustic and radio systems, namely the fact that acoustic transmissions are inherently band-limited. We extend the tone reservation technique to the out-of-band carriers, and design efficient methods for constructing OFDM signals with lower PAR. Two approaches are investigated, one based on a gradient algorithm, and another that uses random sequences. Simulation results show that our techniques can provide PAR reduction without the loss in data rate.**

***Index Terms*—Peak-to-average power ratio, PAR, underwater acoustic communications, OFDM, out-of-band tone insertion.**

I. INTRODUCTION

Multicarrier modulation in the form of orthogonal frequency-division multiplexing (OFDM) has prevailed in recent broadband wireless radio applications due to the low complexity of receivers required to deal with highly dispersive channels. This important trait of OFDM motivates its use in underwater environments (Stojanovic, 2006; Li et al., 2008).

One of the major drawbacks of OFDM modulation is its high peak-to-average power ratio (PAR). Large PARs occur when symbol phases on different carriers line up so as to constructively form peaks in the time-domain signal. Usually, there is a large number of carriers, resulting in large excursions of the signal amplitude. Since the peak transmission power is limited, either by regulatory or hardware constraints, the average power must be reduced, leading to a loss in performance relative to the constant amplitude modulation techniques. Because this loss may outweigh the other advantages of OFDM, the power is often backed-off to a certain degree. The signal will still occasionally exceed the saturation threshold of the power amplifier, resulting in a non-linear distortion.

An OFDM signal is very sensitive to non-linear distortion, which causes spectral growth in the form of inter-modulation products among the carriers. To avoid the non-linear distortion of the power amplifier at transmitter, the input signal to the amplifier should be kept within some limit, carefully selected so as to achieve a balance between the average transmitted power and a distortion that can be tolerated.

Numerous techniques have been developed to tackle the problem of controlling the PAR in an OFDM system (a good overview can be found in Han and Lee, 2005). These techniques have been developed for radio systems, and although they are applicable to acoustic systems as well, there have been no solutions developed specifically for the latter.

In this work we address a PAR reduction technique suited exclusively to underwater acoustic communication systems. The technique is based on inserting a set of tones outside of the nominal transducer bandwidth, so as to cancel the high peaks in the time-domain signal. The out-of-band tones are inserted before the signal is D/A converted and amplified. These tones will subsequently be removed by filtering before transmission. Filtering occurs naturally in the transducer, although additional (analog) filtering is also possible at low frequencies used in typical acoustic systems.[1]

The paper is organized as follows. In Sec. II, the PAR problem is defined. An overview of existing PAR techniques is offered in Section III. The differences between acoustic and radio systems are highlighted in Sec. IV. Sec. V presents the out-of-band tone insertion (OTI) technique, and outlines different algorithms for its efficient implementation. Simulation results are presented in Sec. VI. Sec. VII concludes the paper.

II. PROBLEM STATEMENT

A multicarrier signal is the sum of many independent signals modulated onto carriers of equal bandwidth. In the case of OFDM, these carriers are orthogonal, with spacing $\Delta f = 1/T$, where T is the OFDM symbol duration. The complex baseband representation of an OFDM signal consisting of K carriers is given by

$$x(t) = \sum_{k=0}^{K-1} d_k e^{j2\pi k \Delta f t}, t \in [0, T] \tag{1}$$

The PAR is defined as the ratio between the maximal power and the average power,

$$PAR = \frac{\max_{0\leq t<T}\{|x(t)|^2\}}{\frac{1}{T}\int_0^T |x(t)|^2 dt} \tag{2}$$

[1]Note that acoustic spectrum regulation does not impose explicit requirements on sidelobe suppression.

Since the signal is generated digitally, the PAR can be computed using discrete-time values $x_l = x(lT_s)$, $l = 0, \dots N_s$, where $N_s = Tf_s$ and f_s is the sampling frequency. To accurately account for all the amplitude values, it is necessary to oversample the signal, i.e. f_s has to be higher than the Nyquist rate. An oversampling factor $L = N_s/K = 4$ is considered to be sufficient, since the error due to sampling can be bounded by (Wunder and Boche, 2003)

$$|\max_t |x(t)| - \max_l |x_l|| \leq K[\cos^{-1}(\pi/2L) - 1] \quad (3)$$

OFDM signals are random processes, and their PAR is commonly characterized by the complementary cumulative distribution function (CCDF). The CCDF is defined as the probability that PAR exceeds a certain threshold, $P\{\text{PAR} > \text{PAR}_0\}$. We will use this metric when we discuss the performance of various PAR reduction techniques.

III. Overview of existing techniques

Over the last decades, a number of techniques have been developed for PAR reduction in OFDM radio systems. Below, we briefly summarize the basic principles used in some of these techniques.

Clipping and filtering. This is the simplest technique, in which the signal amplitude is clipped to a predetermined level (ONeill and Lopes, 1995; Li et al., 1998). The distortion caused by clipping is seen as both in-band and out-of-band noise. The latter can be filtered out, but this may cause some peak regrowth, i.e. the signal after clipping and filtering may exceed the clipping level at some point. Repeated clipping-and-filtering iterations reduce the regrowth effect. However, the in-band distortion cannot be reduced by filtering, resulting in performance degradation.

Coding. Coding can also be used to reduce the PAR of an OFDM signal (Jones et al., 1994). For each data block, a codeword with minimal PAR is selected. This approach requires an exhaustive search and storage of codewords in a large lookup table.

Interleaving. Scrambling by a set of interleavers is another technique for PAR reduction (Van Eetvelt et al., 1996; Jayalath and Tellambura, 2000). An interleaver simply permutes the data symbols in a given block. The information in the permuted data block is the same as in the original one, but the resulting waveform in time is different, and may exhibit a lower peak amplitude. Thus, if several interleavers are used, the one that yields the lowest PAR will be chosen. The corresponding signal will be transmitted along with the interleaver's code. The amount of PAR reduction depends on the number of interleavers, but so does the overhead needed to transmit the side information.

Selected Mapping. In this technique, the phases of data symbols are altered in order to avoid the alignment that produces high amplitude peaks (Bauml et al., 1996). The transmitter uses P sets of K phases $\varphi_{k,p} \in [0, 2\pi]$, to generate candidate signals

$$x_p(t) = \sum_{k=0}^{K-1} d_k e^{j\varphi_{k,p}} e^{j2\pi k \Delta f t}, p = 1, \dots P \quad (4)$$

The signal with the lowest PAR is then selected for transmission. This technique also requires transmission of side information to indicate which phase set was used.

Partial Transmitted Sequences. This technique is similarly based on phase manipulations (Muller and Huber, 1997). An input data block of K symbols is partitioned into M disjoint sub-blocks, and the carriers of each sub-block are weighted by phase factors. The phase factors are selected such that the PAR of the resulting signal is minimized. The selection of phases is limited to sets with finite number of elements to keep the search complexity manageable.

Active Constellation. In this technique, outer constellation points in the data block are dynamically extended further out, such that the resulting signal has a lower PAR than the original (Krongold and Jones, 2003). Note that unlike with the previous techniques, this technique requires additional power to regulate the PAR.

Tone Reservation. In this technique, a set of carriers is reserved for control tones which are inserted among the carriers so as to reduce the time-domain peaks (Tellado, 2000). Since the carriers are orthogonal, these additional tones do not cause distortion to the data-bearing carriers. In wireline systems, those carriers whose SNR is too low for reliable information transmission are used for PAR reduction. In wireless systems, it is more difficult to identify the low SNR carriers, since the channel is time-varying, and limited or no feedback may be available from the receiver. Therefore, a set of carriers must be reserved a-priori, resulting in a reduction of the useful information bandwidth. The performance of this scheme depends on the number of control tones and their allocation.

IV. Acoustic vs. radio systems

The major difference between acoustic and radio systems is in the frequency band that they occupy. Acoustic propagation occurs at frequencies that are much lower than those used for typical radio communications (see Fig. 1). The bandwidth is fundamentally limited by absorption, and also by the transducer technology, which imposes strict additional limitations.

It is also interesting to note that unlike radio spectrum, acoustic spectrum usage is not legally regulated, except so as to limit the *maximal* radiated power in a given frequency band and area of operation. However, no strict limitations are imposed on the sidelobe suppression, as it is the case in radio environments where different co-located systems have to be accommodated in adjacent bands. This is not to say that one should create interference to neighboring acoustic systems, but simply that acoustic emissions outside of the nominal bandwidth are left to the designer's best effort. When designing a system, it is also important to keep in mind that higher acoustic frequencies attenuate faster with distance, and,

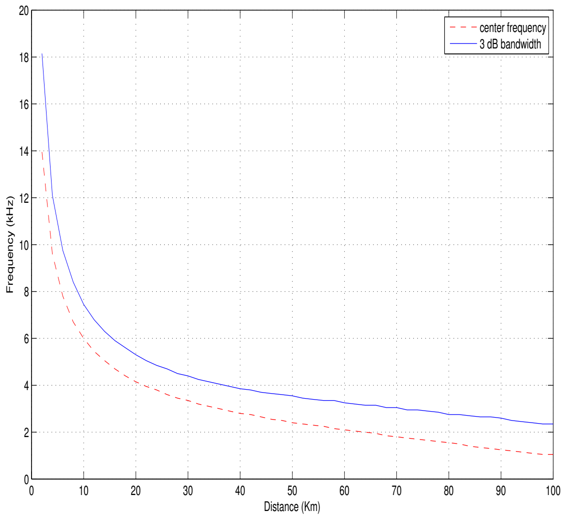

Fig. 1. Optimal center frequency and the 3 dB acoustic bandwidth as functions of distance (Stojanovic, 2006).

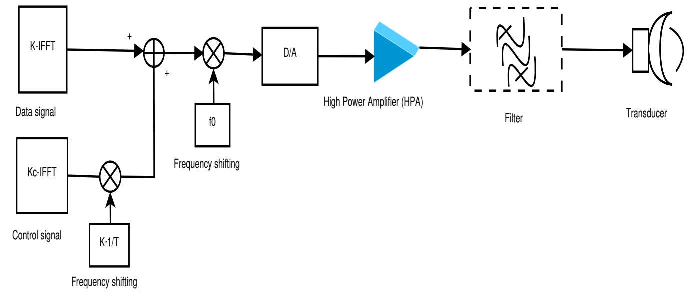

Fig. 2. Block diagram of the transmitter using out-of-band tone insertion.

hence, the interference spectrum measured at the transmitter will not be the same as that measured at the receiver.

V. Out-of-band tone insertion (OTI) technique

The proposed technique is based on adding a data-block-dependent control signal to the original multicarrier signal. The control signal is outside of the useful bandwidth, and is given by

$$y(t) = \sum_{k=0}^{K_c-1} c_k e^{j2\pi(K+k)\Delta f t}, t \in [0, T] \tag{5}$$

where K_c is the number of control tones. The control tones are here placed immediately above the useful bandwidth, but other arrangements are possible as well. The coefficients c_k are chosen so as to reduce the PAR at the input to the non-linear amplifier. The inserted tones are removed after amplification, either by the transducer alone, since it has a limited bandwidth, or by explicit filtering, as illustrated in Fig.2. An efficient implementation of the post-amplifier (analog) filter is deemed possible at frequencies used in typical acoustic communication systems.

The main advantage of the OTI technique is that no side information needs to be transmitted, and, hence, there is no trade-off between the data rate loss and the PAR reduction capability.

Although filtering is applied before transmission, there is some amount of power lost in amplifying the control signal. Therefore, it is important to maintain the number of out-of-band tones as low as possible while aiming for a certain PAR reduction.

While the set of reserved tones is chosen in advance, the coefficients c_k are selected depending on the data vector to be transmitted. These coefficients can be chosen optimally (to minimize the PAR) but the computational demands of optimization are high. We will thus investigate other approaches that are sub-optimal, but offer manageable complexity. Specifically, we want to answer the following questions:
1) Where should the tones be, so that they provide the best performance for all the signals?
2) How many tones are needed in order to achieve a certain improvement?
3) Given a properly chosen number and placement of inserted tones, how can we efficiently compute the coefficients c_k?

A. OTI optimal formulation

Mathematically, we can formulate the problem as

$$\min_{\mathbf{c}} \max_{l=0,\dots N_s-1} |x_l(\mathbf{d}) + y_l(\mathbf{c})| \tag{6}$$

where x_l and y_l are the samples of the information-bearing signal (1) and the control signal (5), which depend on the vector of data symbols $\mathbf{d}$ and the vector of control coefficients $\mathbf{c}$, respectively. The samples of the control signal can also be grouped into a vector,

$$\mathbf{y} = \mathbf{\Phi}\mathbf{c} \tag{7}$$

where $\mathbf{\Phi}$ is the matrix of $N_s \times K_c$ FFT coefficients

$$\phi_{l,k} = e^{j2\pi l(K+k)/N_s}, l = 0, \dots N_s-1, \; k = 0, \dots K_c-1 \tag{8}$$

Denoting by $\mathbf{\Phi}_l$ the l-th row of $\mathbf{\Phi}$, the optimization problem can be expressed as

$$\min_{\mathbf{c}} \max_{l=0,\dots N_s-1} |x_l + \mathbf{\Phi}_l \mathbf{c}| \tag{9}$$

Fortunately, this proves to be a convex problem, which can be solved numerically using quadratically constrained quadratic programming (QCQP) (Tellado, 2000). Namely, since minimizing an absolute value $|a|$ is the same as minimizing its square $p = |a|^2 = aa^*$, the problem can be re-formulated as

$$\min_{\mathbf{c}} p, \text{ subject to } [x_l + \mathbf{\Phi}_l\mathbf{c}][x_l + \mathbf{\Phi}_l\mathbf{c}]^* \le p, \quad \text{for } l = 0, \dots N_s - 1 \tag{10}$$

This formulation involves minimization of a linear function over a set of quadratic constraints, which is a convex problem. In what follows, we will refer to this solution as the optimal solution, and use it as a benchmark to compare the performance of other techniques whose computational demands are conducive to practical implementation. Two such techniques are discussed next.

B. Gradient technique

This technique substitutes for the (optimal) criterion of PAR minimization with the (suboptimal) minimum mean squared error (MMSE) criterion applied to the clipping noise. A gradient algorithm is then used to solve the MMSE optimization in a fast and computationally efficient manner.

To arrive at this algorithm, let us first define the passband signals[2]

$$\begin{aligned} \tilde{x}(t) &= Re\{x(t)e^{j2\pi f_0 t}\} \\ \tilde{y}(t) &= Re\{y(t)e^{j2\pi f_0 t}\} \end{aligned} \tag{11}$$

as well as the composite signal $\tilde{z}(t) = \alpha\tilde{x}(t) + \tilde{y}(t)$, which is input to the power amplifier. The scaling parameter α accounts for setting the maximal signal amplitude to a certain level above the clipping amplitude.

The amplifier non-linearity is modeled as

$$\bar{\tilde{z}}(t) = g[\tilde{z}(t)] = \begin{cases} \tilde{z}(t), & |\tilde{z}(t)| \leq A \\ A \cdot \text{sgn}[\tilde{z}(t)], & |\tilde{z}(t)| > A \end{cases} \tag{12}$$

The resulting error, i.e. the clipping noise, is given by

$$\tilde{e}(t) = \tilde{z}(t) - \bar{\tilde{z}}(t) \tag{13}$$

and the corresponding MSE is defined as

$$D = \int_0^T \tilde{e}^2(t)dt \tag{14}$$

Taking the derivative of D with respect to the control coefficients c_k, we obtain

$$\frac{\partial D}{\partial c_k} = 2\int_0^T \tilde{e}(t)\frac{\partial \tilde{e}(t)}{\partial c_k}dt \tag{15}$$

The integration interval can be split into two complementary parts: $\mathcal{T}$, in which $|\tilde{z}(t)| \leq A$, and $\overline{\mathcal{T}}$, in which clipping occurs. Since the error is zero in the first part, only the second part will contribute to the MSE. In that part, the error is given by $\tilde{e}(t) = \tilde{z}(t) \pm A$, and, hence,

$$\frac{\partial \tilde{e}(t)}{\partial c_k} = \frac{\partial \tilde{e}(t)}{\partial \tilde{z}(t)}\frac{\partial \tilde{z}(t)}{\partial c_k} = \frac{\partial \tilde{z}(t)}{\partial c_k} \tag{16}$$

The remaining term is obtained as

$$\frac{\partial \tilde{z}(t)}{\partial c_k} \equiv \frac{1}{2}\left(\frac{\partial \tilde{z}(t)}{\partial Re\{c_k\}} + j\frac{\partial \tilde{z}(t)}{\partial Im\{c_k\}}\right) = \frac{1}{2}\phi_k^*(t)e^{-j2\pi f_0 t} \tag{17}$$

where

$$\phi_k(t) = e^{j2\pi(K+k\Delta f)t} \tag{18}$$

We thus finally have the MSE gradient,

$$\frac{\partial D}{\partial c_k} = \int_0^T \tilde{e}(t)\phi_k^*(t)e^{-j2\pi f_0 t}dt \tag{19}$$

In this expression, we have switched the integration bounds from $\overline{\mathcal{T}}$ back to the original ones, as this does not affect the result since $\tilde{e}(t) = 0$ outside of $\overline{\mathcal{T}}$.

[2] We will use continuous time for the sake of generality and simplicity of the MMSE analysis, and specialize later to discrete time as it applies to digital processing of the baseband signals.

Further simplification of the above expression is also possible if we express the passband error as

$$\tilde{e}(t) = Re\{e(t)e^{j2\pi f_0 t}\} \tag{20}$$

where $e(t)$ is the complex equivalent evaluated with respect to f_0. Substituting this expression into the gradient (19), the high-frequency terms at $2f_0$ vanish under integration, leaving

$$\frac{\partial D}{\partial c_k} = \frac{1}{2}\int_0^T e(t)\phi_k^*(t)dt \tag{21}$$

The complex envelope $e(t)$ can also be related to an equivalent baseband non-linearity, described by the AM/AM and AM/PM characteristic $g_0[\cdot]$ corresponding to the nonlinearity $g[\cdot]$, such that $e(t) = z(t) - g_0[z(t)]$. For the hard limiter model (12), this function is given by (Jeruchim et al., 1992)

$$\bar{z}(t) = g_0[z(t)] = \begin{cases} z(t), & |z(t)| \leq A \\ \frac{4}{\pi}Ae^{j\arg[z(t)]}, & |z(t)| > A \end{cases} \tag{22}$$

Once the gradient (21) is known, the least mean squares (LMS) algorithm can be applied to calculate the coefficients c_k. The gradient will be calculated in discrete time, giving way to the coefficient update

$$c_k(i+1) = c_k(i) - \mu\sum_{l=0}^{N_s-1} e_l(i)\phi_{l,k}^* \tag{23}$$

where μ is the step size, $e_l(i)$ is the clipping error in the i-th iteration, and $\phi_{l,k}$ are as given in the expression (8). Using the notation of Sec.V-A, the vector update is given by

$$\mathbf{c}(i+1) = \mathbf{c}(i) - \mu\sum_{l=0}^{N_s-1} e_l(i)\mathbf{\Phi}_l' \tag{24}$$

Instead of generating the control signal from the coefficients after the algorithm has converged, the signal itself can be updated directly (Tellado, 2000). The LMS algorithm will then operate in the time domain, generating the signal vector $\mathbf{z} = \mathbf{x} + \mathbf{y}$. The corresponding update equation is obtained by multiplying both sides of the expression (24) by the FFT matrix $\mathbf{\Phi}$, and adding the information bearing signal:

$$\mathbf{z}(i+1) = \mathbf{z}(i) - \mu\sum_{l=0}^{N_s-1} e_l(i)\mathbf{\Phi}\mathbf{\Phi}_l' \tag{25}$$

Since the error depends only on the signal, $e_l(i) = z_l(i) - \bar{z}_l(i)$, the control coefficients never need to be computed explicitly. The vectors $\mathbf{\Phi}\mathbf{\Phi}_l'$, $l = 0, \ldots N_s - 1$, can be pre-computed and stored, which accounts for the very low computational complexity of the algorithm. The algorithm is initialized by $\mathbf{z}(0) = \alpha\mathbf{x}$.

While the signal *shape* cannot be controlled, its range can be controlled through the parameter α. Namely, the scaling parameter can be selected for each incoming block independently, to suit the maximal amplitude x_{max} of that block. We propose to choose the scaling parameter α so that the clipping level A is initially exceeded by no more than a pre-determined amount Δ. In other words, the scaling parameter is set for each new block as

$$\alpha = \begin{cases} \Delta \cdot A/x_{max}, & x_{max} > \Delta \cdot A \\ 1, & x_{max} \le \Delta \cdot A \end{cases} \quad (26)$$

The impact of the threshold Δ on the convergence time and on the PAR reduction was studied through simulation, whose results will be discussed in Sec.VI.

The algorithm is summarized below.

Preparation: These steps only need to be executed once.

- Select a desired threshold Δ.
- Choose the set of inserted tones K_c.
- Compute and store the kernel vectors $\mathbf{\Phi\Phi}'_l$ for all $l = 0, \ldots, N_s - 1$.
- Specify the maximum number of algorithm iterations, I.

Run time: These steps are executed for each OFDM block that exceeds the clipping level.

- Set the iteration index to $i = 0$.
- Find $x_{max} = max_{l=0,\ldots N_s-1}\{|x_l|\}$, and set the scaling parameter $\alpha = \Delta \cdot A/x_{max}$. Set $\mathbf{z}(0) = \alpha\mathbf{x}$.
- Set $\mathbf{e}(0)$ to an arbitrary non-zero value.
- While $i < I$ or $||\mathbf{e}(i)||^2 > 0$:
  - Apply the clipping rule (22) to $\mathbf{z}(i)$ to obtain $\bar{\mathbf{z}}(i)$.
  - Compute the error vector $\mathbf{e}(i) = \mathbf{z}(i) - \bar{\mathbf{z}}(i)$.
  - Compute $\mathbf{z}(i+1)$ according to (25).
  - Increment the iteration counter.
- Generate the samples of the modulated signal $\tilde{z}(t) = Re\{z(t)e^{j2\pi f_0 t}\}$, and pass them on to the D/A converter. Note that this process may involve upsampling of the elements z_l of the vector $\mathbf{z}$ obtained from the previous step, so that sufficient sampling frequency is ensured for the passband digital signal, which is fed to the D/A converter and from there on to the amplifier and transducer.

The LMS convergence time, nominally $20N_s$ iterations, amounts to $80 \cdot K$ iterations with the oversampling ratio of 4. These iterations need to be completed within the duration of one block, $T = K/B$, in order for a real time implementation to be possible. If we take as an example a 160 MHz processor, this will be possible so long as 160 MHz/80B=2/B[MHz] is greater than the number of instructions required per iteration. As we shall see from the simulation results, it suffices to perform only a few iterations if the initial conditions are chosen carefully, which can be accomplished through a proper selection of the threshold Δ.

C. Random insertion

In this technique, the out-of-band tones are generated from a finite modulation alphabet, which can be the same as that of the information-bearing signal, or different. Thus, there is a finite number of possible selections for the control sequence, but this number may be large (M^{K_c} for the modulation level M), making it impractical to conduct an exhaustive search.

Instead of performing a systematic search, the selection is made from a finite set of randomly generated control sequences. The search for the best sequence is conducted until a certain improvement in the PAR is reached, or until a predetermined number of trials have been exhausted (after which the best candidate sequence is retained).

Random tone insertion aims to reduce the implementation complexity by sacrificing some improvement in the PAR reduction capability. This technique is similar to interleaving (Jayalath and Tellambura, 2000) in that the transmitter only needs a random generator for the out-of-band tones and a module that computes the PAR. Note, however, that the two techniques are conceptually different, and can even be combined.

Two specific questions are to be addressed with this technique. The first question refers to the size of the control sequence alphabet. The larger the alphabet, the better the PAR reduction (the optimal solution described in Sec.V-A can in fact be regarded as a modulation with an infinite constellation size). However, an increase in the modulation alphabet implies a greater number of candidate sequences, which complicates the search. The second question refers to the number of trials needed to achieve a certain performance. Obviously, the more trials, the better, but we would like to know how much can the performance be improved with a relatively small number of trials, suitable for a practical implementation. These questions will be addresses in the following section.

VI. SIMULATIONS RESULTS

A simulation analysis was conducted for an OFDM signal with 512 carriers employing QPSK in the 8-28 kHz band. A total of 10,000 randomly generated OFDM blocks were used to assess the performance of the proposed techniques. The results are contrasted with the original signals' statistics (no PAR reduction method employed) and the optimal case, which was evaluated according to the principles of Sec.V-A.

A. Control bandwidth allocation

The out-of-band tones can be placed either below or above the useful bandwidth, and the question is which is better. Fig. 3 shows the system performance under different allocation policies. Indicated in the figure is the bandwidth occupied by the control signal consisting of 64 tones. Clearly, allocating the control signal above the useful bandwidth results in a better performance. Note that such a placement is also advantageous from the viewpoint of radiated out-of-band acoustic power, since higher frequencies attenuate faster with distance. The control tones are best placed as close as possible to the upper edge of the data bandwidth.

B. Number of control tones

Given the control signal placement at the upper edge of the useful bandwidth, we now want to determine the minimum number of control tones needed to ensure a certain PAR reduction. Note that there is a trade-off here, as more tones enable better control of the peak power, but increase the overall average power. The relationship between the number of control tones and the achievable improvement is depicted in Fig. 4.

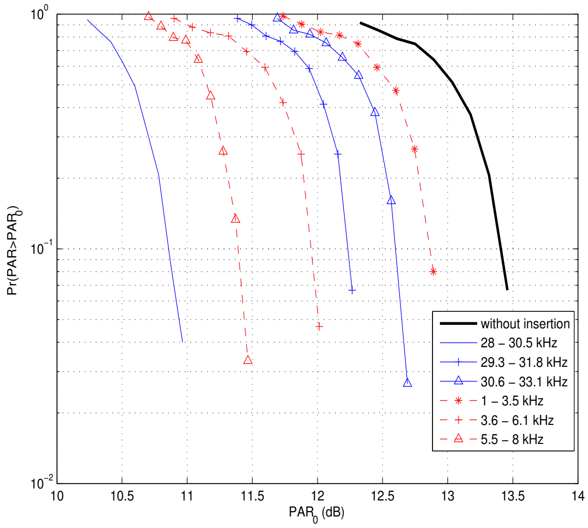

Fig. 3. CCDF of the PAR when control tones are inserted above the useful bandwidth (solid) and below the useful bandwidth (dashed). Legend indicates the bandwidth occupied by the control signal.

The average achievable improvement is defined as the average PAR reduction achieved using the optimal control signal. This result quantifies the effect of diminishing returns that takes place as the number of control tones is increased. More importantly, it demonstrates that an improvement of several dB is available from the OTI technique with a reasonably small number of control tones.

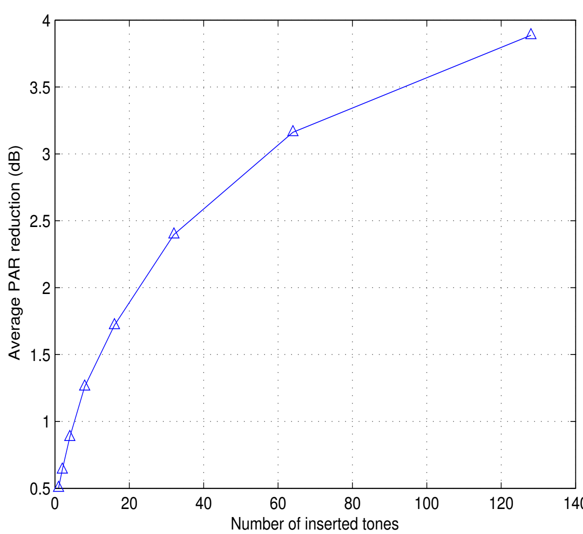

Fig. 4. PAR improvement vs. the number of out-of-band tones.

In what follows, we address the performance of practical OTI techniques, namely the gradient technique and the random insertion technique.

C. OTI–Gradient technique

The gradient algorithm described in Sec.V-B was applied to each incoming data block, scaled in accordance with a pre-specified threshold Δ. The step size was set to $\mu = 2/K$, and the threshold Δ was set to 4 dB. This value of the threshold was chosen through a preliminary analysis, whose results are shown in Fig. 5. This figure shows the average PAR reduction as a function of the threshold Δ. Clearly, there exists an optimal value of Δ for which the PAR improvement is maximized.

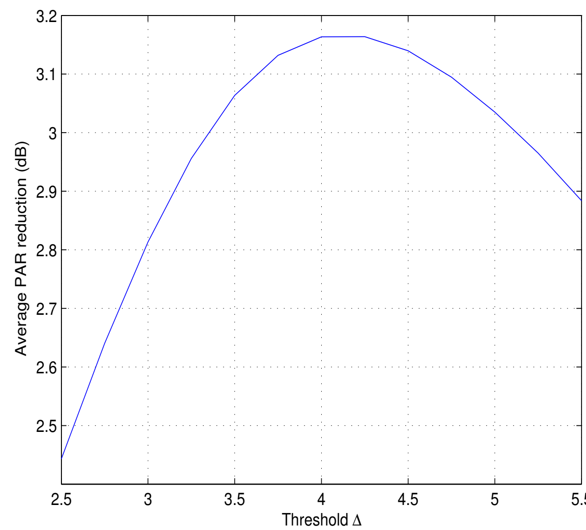

Fig. 5. Average PAR reduction as a function of the threshold Δ.

Fig. 6 shows the performance of the gradient technique, obtained after a varying number of LMS iterations. The PAR reduction achieved after the first few iterations is outstanding, with diminishing improvement thereafter. For example, a 2.5 dB reduction is obtained after only three iterations when the original PAR is 13 dB.

Performance of the gradient technique is further illustrated in Fig.7, which shows the normalized MSE, i.e. the variance of the clipping error obtained after a given number of iterations, averaged over all data blocks. These results demonstrate that a considerable improvement is available from the OTI technique at a very low computational cost.

D. OTI–Random insertion

Fig. 8 shows the results obtained using the random insertion technique described in Sec.V-C. The maximal number of trials (randomly generated control sequences) is set to 100, and the modulation method (alphabet size) used for the control signal is varied. The data sequence is modulated using QPSK. Interestingly, there is not much to be gained by increasing the modulation level from 2 to 16. This fact justifies the use of simple control sequences, such as BPSK or QPSK. We

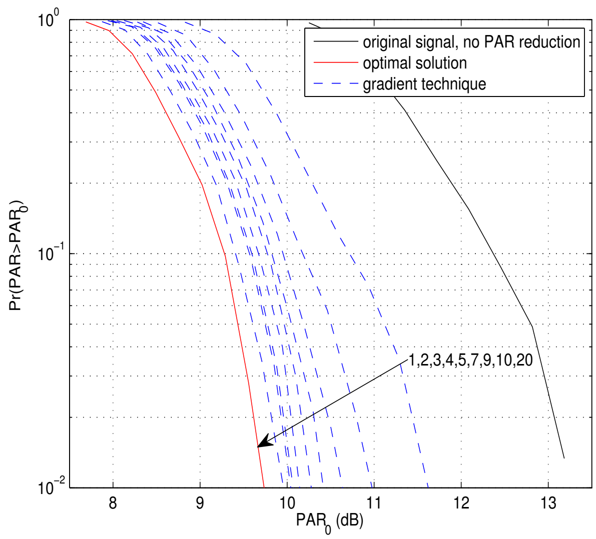

Fig. 6. CCDF of the PAR resulting from the gradient technique after a varying number of iterations.

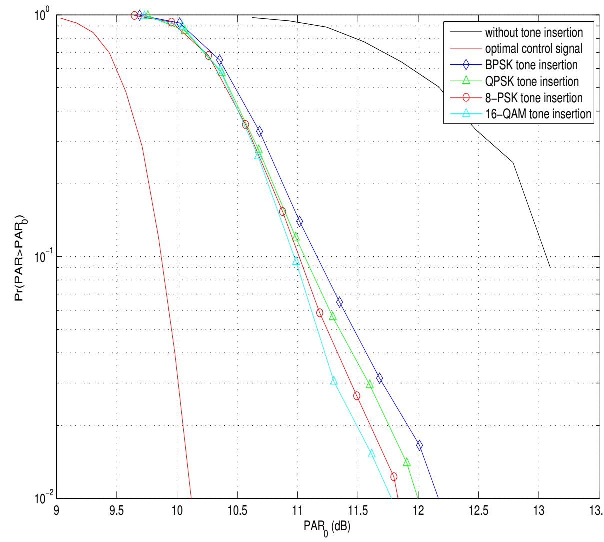

Fig. 8. CCDF of the PAR resulting from random insertion technique. The data sequence is modulated using QPSK, and the number of trials is limited to 100.

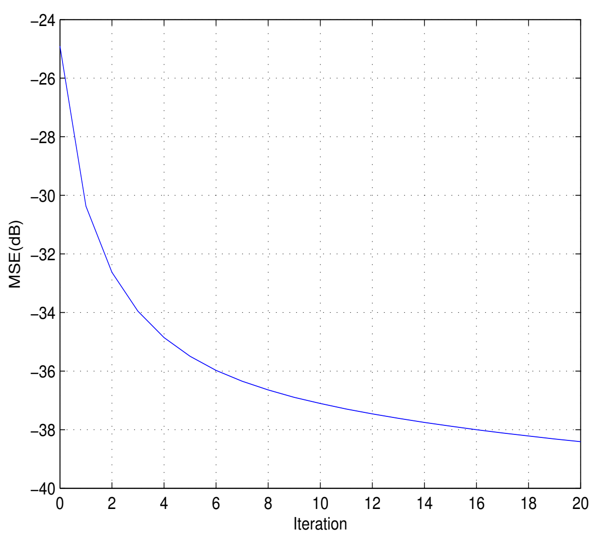

Fig. 7. Normalized MSE of the gradient technique.

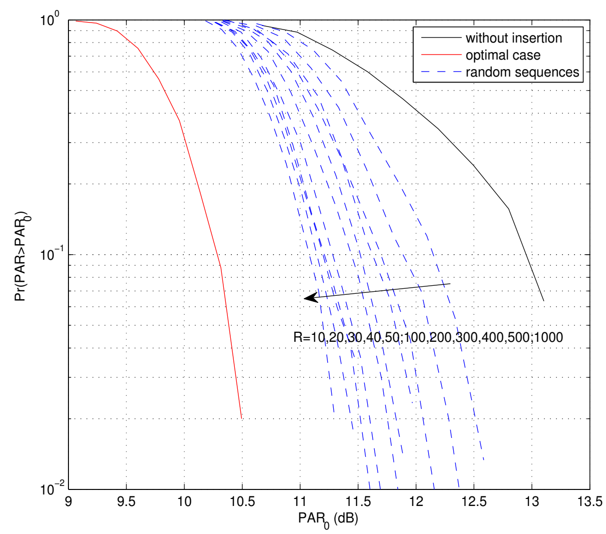

Fig. 9. CCDF of the PAR resulting from random insertion technique with a varying number of trials. The data and the control sequence are modulated using QPSK.

also note that the overall PAR reduction is comparable to that obtained using the gradient technique.

The question of the number of trials needed to achieve a certain performance is addressed in Fig.9. Similarly as with the number of iterations in the gradient technique, we observe an effect of diminishing returns with the number of trials. However, the results are somewhat less encouraging, since at least a few tens of trials are needed to achieve a substantial improvement. At a (hypothetical) 1000 trials, the performance deviates from the optimal by about 0.75 dB. In comparison, the gradient technique achieves this in about ten iterations.

VII. Conclusions

Out-of-band tone insertion is proposed as a PAR reduction technique for underwater acoustic OFDM systems. A set of tones is inserted outside of the nominal signal bandwidth prior to D/A conversion. The control tones are digitally optimized to provide PAR reduction before the signal is D/A converted and fed to the (nonlinear) power amplifier. The tones are subsequently removed by the transducer which acts as a filer, or by explicit filtering. The main advantage of this technique is that PAR improvement comes at no reduction in the data rate.

Two approaches were considered for computationally-efficient design of the control signal: a gradient technique which minimizes the mean-squared clipping error, and a random insertion technique in which the selection of control signal is made from a finite set of randomly generated symbols. The performance of these techniques, as well as the number and placement of control tones, were studied via a numerical analysis.

Results show that the best tone placement is at the high end of the useful signal bandwidth. The PAR reduction grows with the number of tones, but there is an effect of (exponentially) diminishing returns, which justifies the use of a relatively small number of tones (not more than what is used for the information signal). Both the gradient technique and the random insertion technique offer non-negligible PAR improvements. The gradient technique exhibits fast convergence, yielding a close-to-optimal solution in only a few LMS iterations. Random insertion offers a comparable, albeit slightly inferior performance, using control symbols from the same alphabet as the data symbols (QPSK), and a search limited to about a hundred sequences.

Future work in this area should target the design of low-complexity systematic search methods for the random insertion technique, as well as integration of the OTI principle with other PAR reduction techniques such as interleaving or phase randomization techniques.

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