Information Rates of Energy Harvesting Communications with Intersymbol Interference

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***Abstract*—We consider energy harvesting communications over an intersymbol interference (ISI) channel corrupted by additive white Gaussian noise. We assume that the energy arrivals at the transmitter, which is equipped with a finite battery, are independent and identically distributed, and provide a method of computing achievable information rates with channel inputs selected from a finite alphabet. We illustrate the methodology via a set of examples quantifying the effects of energy arrival rates as well as battery capacity on the achievable information rates.**

I. Introduction

Energy harvesting (EH) communications have been receiving attention from a physical layer perspective with the motivation of designing self-sustaining communication systems that do not need fixed batteries. In addition to earlier works on energy/power scheduling type solutions (e.g., [1], [2]), there are also efforts to characterize channel capacity for different cases in the recent literature. In [3], it is shown that the capacity of an EH communication system with a transmitter equipped with an infinite battery operating over an additive white Gaussian noise (AWGN) channel is equivalent to that of an AWGN channel with an average power constraint equal to the average recharge rate. A similar result for the infinite battery case can also be obtained for the discrete-input discrete-output channel models. It is also elementary to consider the case of no battery at the transmitter for both discrete channels as well as AWGN formulations. However, computing the channel capacity for the case of finite battery with stochastic energy arrivals is much more challenging.

The authors in [4] provide a capacity expression for energy harvesting communications (for a finite battery capacity) using the formulation in [5]. They consider both discrete memoryless channels as well as channels with discrete inputs and continuous outputs, such as binary input AWGN channels. However, the capacity expression is not amenable for computation. The authors also conjecture that the optimal transmission policy needs to use Shannon strategies based on the current battery level only, which reduces the system to a finite state model. With this conjecture, it is possible to evaluate the resulting information rates for different inputs (e.g., independent and identically distributed (i.i.d.) or Markov), or even optimize the Markov input distribution.

For the special case of i.i.d. energy arrivals, unit battery and a noiseless channel, the EH communication system is equivalent to a timing channel with transmit-side causal channel state information, and a single-letter expression on its capacity as well as computable upper and lower bounds are developed in [6]. An approximation of the channel capacity within a constant gap is derived in [7] under some general conditions. In addition to these information theoretic developments, there have also been recent efforts on explicit channel coding/modulation solutions for energy harvesting communication systems [8], [9].

In the current literature, the focus of EH communications research has been on discrete memoryless or AWGN channel formulations. With the motivation that there are many wireless communication systems for which the channel is modeled as frequency selective resulting in intersymbol interference (ISI), it is also of significant interest to study the case of ISI channels with AWGN. Such channel models are highly suitable for underwater acoustic (UWA) systems as well [10].

Our objective in this letter is to extend the available information-theoretic results on achievable information rates for EH communications to the case of ISI channels. Specifically, we consider an EH transmitter sending data over an ISI channel with AWGN with i.i.d. inputs drawn from a finite signal constellation for which one of the symbols (denoted by “0”) consumes no energy in transmission. A special case is on-off signaling. We assume that the transmitter has causal knowledge of the energy arrivals; however, the receiver does not. Even though the intended channel inputs are drawn independently, transmission is constrained by the availability of energy in the battery, i.e., the transmitter is limited to sending the “0” symbol if the battery is empty. Assuming an i.i.d. energy arrival process and a finite battery, we provide a method of computing achievable information rates for reliable transmission. The approach is based on a suitable application of the forward recursion of the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm, which has previously been used in estimating the information rates over ISI channels (without any transmitter energy limitations) [11], [12]. The method can be extended to the case of Markov channel inputs as well as Markov energy arrivals in a straightforward manner.

The letter is organized as follows. We provide the system model in Section II. We detail the achievable rate calculation process in Section III. We evaluate the information rates for several examples in Section IV, and illustrate the effects of energy arrival rates as well as ISI and noise levels on the reliable transmission rates. Finally, we conclude the letter in Section V.

II. System Model

We consider an EH communication system for which the transmitter is equipped with a finite battery and the energy arrivals are i.i.d. Bernoulli random variables in each symbol interval. That is, in each symbol interval, either no energy arrives (with probability $1-\alpha$), or a unit energy arrives (with

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probability α). We refer to α as the average energy arrival rate. The transmitter battery is of capacity B (i.e., it can store B units of energy). Note that it is also possible to consider other energy arrival models, e.g., more than one unit arriving in each symbol interval, and/or arrivals having memory.

The EH communication system model is depicted in Fig. 1. A message W is transmitted using N channel symbols denoted by the vector $\boldsymbol{X} = [X_1, X_2, \ldots, X_N]$. The channel output is denoted by $\boldsymbol{Y} = [Y_1, Y_2, \ldots, Y_N]$, and used by the decoder to produce an estimate of the message, denoted by $\hat{W}$. We assume that the X_i's are drawn from an $M-$ary modulation scheme where one of the symbols consumes no energy. We denote by β_j the probability of the $j-$th input symbol m_j, $j = 0, 1, \ldots, M-1$. The symbol with zero-energy cost is denoted by m_0. A special case if on-off signaling where the on-signal consumes unit energy. While different energy arrival models are possible, we assume that the harvested energy is first stored in the battery, and it becomes available for use in the next symbol interval as in the model employed in [2]. A non-zero symbol may only be transmitted if there is energy in the battery at the beginning of the symbol interval.

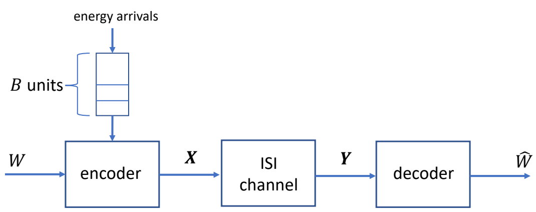

Fig. 1: EH communication over an ISI channel.

Transmission takes place over an L-tap ISI channel with AWGN, i.e., the received signal in the n-th symbol interval is given by

$$Y_n = \sum_{i=0}^{L-1} h_i X_{n-i} + Z_n \tag{1}$$

where h_i's are the ISI channel coefficients, and Z_n's are the AWGN noise terms with zero mean and variance σ^2. Our objective is to compute achievable transmission rates with i.i.d. channel inputs as described in the next section.

III. Achievable Rates with EH Transmitters over ISI Channels

A. General Approach

Achievable information rates with EH transmitters for reliable transmission over an ISI channel is given by the limit

$$R = \lim_{N \to \infty} \frac{1}{N} I(\boldsymbol{X}; \boldsymbol{Y}), \tag{2}$$

where the channel inputs are i.i.d. as described in the previous section. Our approach to evaluating this limit and determining the transmission rate R is based on simulation as in the computation of the information rates with finite alphabet inputs over ISI channels without any energy constraints at the transmitter (e.g., as done in [11], [12]). In other words, we simulate a long realization of the channel input and output processes (driven by the energy arrival process with the given statistics), estimate the mutual information $I(\boldsymbol{X}; \boldsymbol{Y})$, and divide the result by the number of channel uses N. The approach is justified by the Shannon-MacMillan-Brieman theorem which applies to our present set-up. Furthermore, the error in estimating R is $O(1/\sqrt{N})$ [13], i.e., by taking a large number of realizations any desired accuracy can be guaranteed.

To describe the algorithm in more detail, let us write

$$\begin{aligned} I(\boldsymbol{X}; \boldsymbol{Y}) &= H(\boldsymbol{Y}) - H(\boldsymbol{Y}|\boldsymbol{X}) & (3)\\ &= H(\boldsymbol{Y}) + H(\boldsymbol{X}) - H(\boldsymbol{X}, \boldsymbol{Y}) & (4)\\ &= NH(\beta_0, \ldots, \beta_{M-1}) + H(\boldsymbol{Y}) - H(\boldsymbol{X}, \boldsymbol{Y}) & (5) \end{aligned}$$

where the last line follows from the independence of the channel inputs. The problem is thus transformed into computation of the entropy of output sequence and the joint entropy of input and output sequences, which are given by $H(\boldsymbol{Y}) = E[-\log p(Y_1, Y_2, ..., Y_N]$ and $H(\boldsymbol{X}, \boldsymbol{Y}) = E[-\log p(X_1, X_2, \ldots, X_N, Y_1, Y_2, ..., Y_N)]$, where $p(.)$ stands for the joint density. In other words, as an ingredient of the simulation based approach, we need to compute the joint density of $\boldsymbol{Y}$ and $[\boldsymbol{X}, \boldsymbol{Y}]$ at specific values of the input and output vectors, denoted by $\{x_1, x_2, \ldots, x_N\}$ and $\{y_1, y_2, \ldots, y_N\}$, respectively.

B. Joint Density of the Output Sequence

Let us now focus on the computation of $p(y_1, y_2, \ldots, y_N)$ for a given sequence of channel output realizations. To do so, we define the forward recursion variable

$$\alpha_k(j) = p(y_1, y_2, \ldots, y_N, S_k = j), \tag{6}$$

where $k = 1, 2, \ldots, N$ denotes the time index, and S_k is the state variable denoting the battery level and the ISI channel state at time k. The state of the system is determined by the $B+1$ battery levels and the previous $L-1$ channel inputs, hence there are $(B+1) \times M^{L-1}$ possible states. Clearly, we can obtain the desired joint density term by simply adding the $\alpha-$ variables at time N over all possible states.

For the $\alpha-$ variables, we can write

$$\begin{aligned} \alpha_k(j) &= p(y_1, y_2, \ldots, y_k, S_k = j) & (7)\\ &= \sum_i p(y_1, y_2, \ldots, y_k, S_k = j, S_{k-1} = i) & (8)\\ &= \sum_i p(y_1, y_2, \ldots, y_{k-1}, S_k = j, S_{k-1} = i) & \\ &\qquad p(y_k | y_1, \ldots, y_{k-1}, S_k = j, S_{k-1} = i) & (9)\\ &= \sum_i p(y_1, y_2, \ldots, y_{k-1}, S_{k-1} = i) & \\ &\qquad P(S_k = j | S_{k-1} = i) p(y_k | S_k = j, S_{k-1} = i) & (10) \end{aligned}$$

where the last line follows from the fact that conditioning on the battery and channel state at time $k-1$, the state at time k is independent of the previous channel outputs, and conditioned on the previous and current states, the past outputs are independent of the channel output at time k.

Defining the state transition variable at time k as

$$\gamma_k(i, j) = p(y_k | S_k = j, S_{k-1} = i) P(S_k = j | S_{k-1} = i), \tag{11}$$

we obtain a recursion for computing the $\alpha-$variables as

$$\alpha_k(i) = \sum_i \gamma_k(i, j) \alpha_{k-1}(i). \tag{12}$$

Noting that it is easy to compute $\gamma_k(i, j)$ using the AWGN channel statistics and the input distribution for all possible state transitions, and using the initialization $\alpha_0(1) = 1$ and $\alpha_0(j) = 0$ for $j \neq 1$ (where the state S_1 corresponds to empty

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battery state and clear channel), we can obtain all the $\alpha_k(j)$ terms, and hence the joint density $p(y_1, y_2, \ldots, y_N)$. This joint density is then used to accurately estimate the joint entropy of the output sequence (using a sufficiently large N).

C. Joint Density of the Input and Output Sequences

In a similar fashion, we can also compute the joint density of input and output sequences at their simulated values, i.e., $p(x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N)$, from which $H(\boldsymbol{X}, \boldsymbol{Y})$ will be estimated. In this case, we define

$$\alpha'_k(j) = p(x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N, S_k = j) \quad (13)$$

as the forward recursion variable, and following an analogous line of reasoning as before, we obtain

$$\alpha'_k(i) = \sum_i \gamma'_k(i,j)\alpha'_{k-1}(i). \quad (14)$$

with $\gamma'_k(i,j) = p(x_k, y_k | S_k = j, S_{k-1} = i)P(S_k = j | S_{k-1} = i)$, which can also be written as

$$\gamma'_k(i,j) = P(x_k)P(S_k = j|S_{k-1} = i)p(y_k|S_k = j, S_{k-1} = i). \quad (15)$$

With a similar initialization as in the computation of $\alpha_k(j)$ terms, we can compute $\alpha'_k(j)$ for all k and j values, from which we obtain the desired joint density as

$$p(x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N) = \sum_j \alpha'_N(j). \quad (16)$$

The joint entropy $H(\boldsymbol{X}, \boldsymbol{Y})$ can then be estimated by scaling the negative logarithm of the joint density by the number of channel realizations as accurately as desired by running a sufficiently long simulation.

D. Selection of the Input Distribution

It is also of interest to compute the optimal values of the input probabilities to obtain the highest achievable information rate for a given system. There is no simple solution for this optimization problem for the general case of finite battery, ISI and additive noise; however, it is possible to utilize the optimal input distribution for the ideal scenario of no noise.

For the case of no battery (with no ISI or noise), the EH communication system becomes equivalent to a discrete memoryless channel, the optimal input distribution can be found numerically. In some cases, and the optimization problem can even be solved analytically. For instance, for on-off signaling, with an energy arrival probability of α, we simply have an asymmetric binary-input binary-output channel (with 0 to 0 transition probability of 1, 1 to 1 transition probability of α), and the optimal value of $\beta = P(X = 1)$ can be determined to be

$$\beta = \frac{\exp(\frac{1-\alpha}{\alpha}\ln(1-\alpha))}{1 + \alpha\exp(\frac{1-\alpha}{\alpha}\ln(1-\alpha))}. \quad (17)$$

As another illustration, assuming on-off signaling and taking $\beta_j = \beta$, in Fig. 2, we depict the achievable rates for different battery capacities as a function of the input ones' density β for an energy arrival probability of $\alpha = 0.5$. The optimal values of the input ones' density are approximately 0.3, 0.34, 0.35, 0.37, 0.38 and 0.4, with the corresponding information rates of 0.59, 0.76, 0.84, 0.88, 0.91 and 0.93 bits/channel use for battery capacities of 1, 2, ..., 6, respectively. We observe that when the battery capacity is larger, the probability of an empty battery is reduced, and the EH transmitter can employ input densities closer to 0.5 (which is optimal if there is no energy shortage). It is also clear that symmetric input distributions are highly suboptimal, and selection of the parameter β is critical.

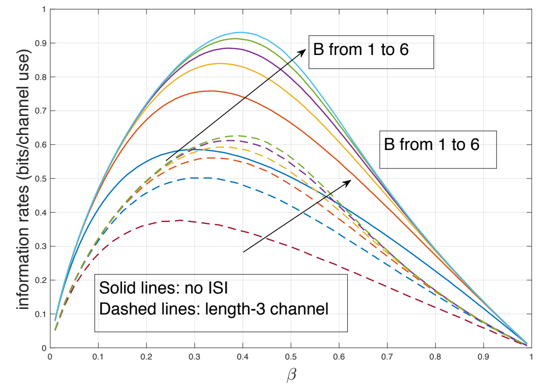

Fig. 2: Information rates as a function of the input ones' density for different battery sizes ($\alpha = 0.5$).

Fig. 2 also depicts the information rates for the ISI channel $\frac{1}{\sqrt{3}}[1\ 1\ 1]$ as a function of the input ones' density for $\sigma^2 = 0.1$ (again for $\alpha = 0.5$). As in the case with no ISI, the need for using asymmetric inputs is clear. It is also seen that the β values that optimize the information rates for the previous case are nearly optimal for this scenario as well.

Remark 1. By using different probabilities of 1 at the input for different battery states (recall that the state is available causally at the transmitter), higher transmission rates can be obtained. However, this optimization is numerically more consuming, and it is not performed here.

Remark 2. For the case of unit battery and no ISI, using the optimal value of input ones' density, we are essentially computing the rates achievable by optimal i.i.d. Shannon strategies (see [6]). For higher battery capacities and with ISI, however, using a single β value (independent of the battery and the channel state) is *suboptimal*. In this case, the scheme can be considered as Shannon strategies based on *partial* state information (i.e., information on "empty" vs. "non-empty" battery).

Remark 3. It is possible to obtain increased rates using Markov inputs (for which the developed approach can be used with slight modifications in the definition of the system states). It is shown in [4] that the gains with Markov inputs are very small, and i.i.d. codebooks are near optimal, for the case of no ISI. We expect that i.i.d. codebooks are near optimal for the case with ISI as well.

IV. Numerical Examples

We now provide several numerical results comparing the achievable information rates of EH communications. Specifically, we consider on-off signaling with channel inputs 0 and 1. The input ones' densities are selected as those that optimize the information rates for the case of no ISI and no noise.

In Fig. 3, we depict the information rates as a function of the inverse of the noise variance for the case with no battery for four different channels (given in the caption of the figure) when $\alpha = 0.5$. Clearly, the information rates increase with reduced noise variance, saturating at the value obtained for the no noise scenario. It is also observed that there is a

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loss in the information rate for an EH communication system when the ISI becomes more severe, which is similar to the behavior observed for communication systems without the EH constraints.

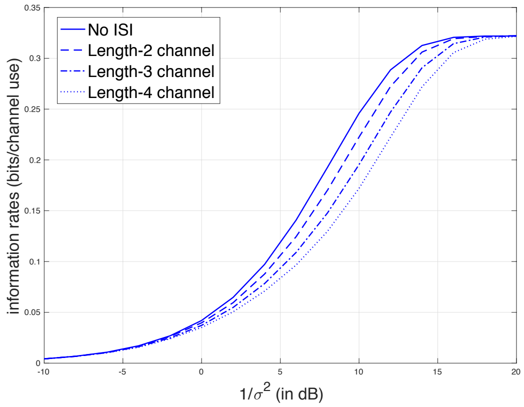

Fig. 3: Comparison of information rates with $\beta = 0.4$, $\alpha = 0.5$ and $B = 0$. The channels are 1) no ISI, 2) $\frac{1}{\sqrt{2}}[1\ 1]$ ISI channel, 3) $\frac{1}{\sqrt{3}}[1\ 1\ 1]$ ISI channel, and 4) $\frac{1}{2}[1\ 1\ 1\ 1]$ ISI channel.

As a second example, in Fig. 4, we provide the achievable rates with two different ISI channels (length two and length four channels used above) for different battery capacities (for $\alpha = 0.5$). We observe that there is a significant increase in the achievable rates when the battery capacity is increased from 1 to 2, then to 3. However, the gains become marginal with larger B values. This behavior is due to the use of on-off signaling; with higher order modulations, the amount of increase will be significant until we reach a larger battery capacity, becoming marginal at higher B values. It is particularly noteworthy that even a small battery improves the system performance significantly (comparing these results with those in Fig. 3).

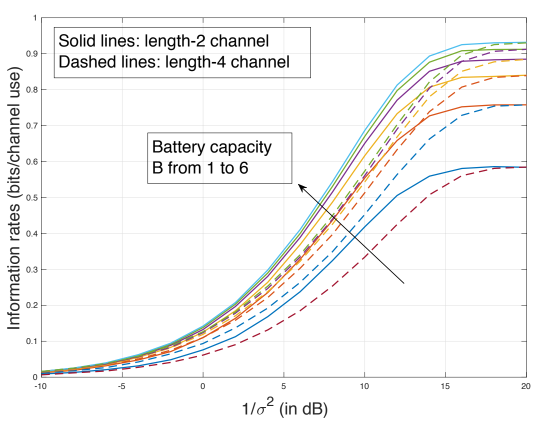

Fig. 4: Information rates of 2 and 4-tap channels in the previous example for different battery capacities.

Finally, in Fig. 5, we illustrate the achievable rates for three different energy arrival probabilities $\alpha = 0.2, 0.5$ and 0.8. We consider two different battery sizes: $B = 1$ and $B = 6$, and two different channels: no ISI and length-four ISI channel with equal gain taps. The optimal ones' densities for $B = 1$ are $0.18, 0.3$ and 0.4, and for $B = 6$ are $0.19, 0.4$ and 0.5, corresponding to $\alpha = 0.2, 0.5$ and 0.8, respectively. The results clearly quantify the role of the energy arrival rates as well as the battery sizes for EH communications over ISI channels.

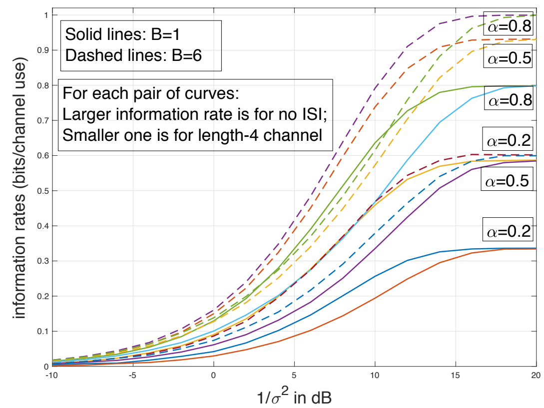

Fig. 5: Information rates for three different energy arrival rates.

V. Conclusions

We have provided a way of computing achievable rates of EH communication systems with i.i.d. inputs drawn from a finite alphabet over ISI channels by extending the previously known information rate estimation approaches for the case with no energy constraints. This tool allows us to quantify the effects of energy arrival rates, battery capacities, specific ISI patterns and noise levels on the reliable transmission rates. The results indicate importance of the input distribution optimization, and that even a battery of small capacity helps improve the rates considerably.

References

[1] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: optimal policies," IEEE JSAC, vol. 29, pp. 1732-1743, Sep. 2011.
[2] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," IEEE Trans. on Wireless Comm., vol. 11, pp. 1180-1189, Mar. 2012.
[3] O. Ozel and S. Ulukus, "Achieving AWGN capacity under stochastic energy harvesting," IEEE Trans. on Inf. Theory, vol. 58, pp. 6471-6483, Oct. 2012.
[4] W. Mao and B. Hassibi, "Capacity analysis of discrete energy harvesting channels," IEEE Trans. on Inf. Theory, vol. 63, pp. 5850-5885, Sep. 2017.
[5] S. Verdu and T. S. Han, "A general formula for channel capacity," IEEE Trans. on Inf. Theory, vol. 40, no. 4, pp. 1147-1157, July 1994.
[6] K. Tutuncuoglu, O. Ozel, A. Yener and S. Ulukus, "Binary energy harvesting channel with unit-sized battery," IEEE Trans. on Inf. Theory, vol 63, pp. 4240-4256, July 2017.
[7] Y. Dong, F. Farnia and A. Ozgur, "Near optimal energy control and approximate capacity of energy harvesting communication," IEEE JSAC, vol. 33, pp. 540-557, Mar. 2015.
[8] M. Dabirnia and T. M. Duman, "Code design for binary energy harvesting channel," IEEE International Symposium on Information Theory (ISIT), pp. 1127-1131, Aachen, Germany, June 2017.
[9] M. Ozates and T. M. Duman, "Channel coding for energy harvesting communications using run length limited codes," IEEE GLOBECOM, Abu Dhabi, UAE, December 2018.
[10] M. Stojanovic and J. Presig, "Underwater acoustic communication channels: propagation models and statistical characterization," IEEE Communications Magazine, pp. 84-89, Jan. 2009.
[11] D. Arnold and H.-A. Loeliger, "On the information rate of binary-input channels with memory," in Proc. IEEE International Conf. Commun. (ICC), vol. 9, Helsinki, Finland, June 2001, pp. 2692-2695.
[12] H. D. Pfister, J. B. Soriaga and P. H. Siegel, "On the achievable information rates of finite state ISI channels," in Proc. IEEE GLOBECOM, vol. 5, San Antonio, TX, Nov. 2001, pp. 2992-2996.
[13] T. M. Duman and J. Zhang, "A note on convergence rate of constrained capacity estimation algorithms over ISI channels," (invited), 3rd Information Theory and Applications (ITA) Workshop, 4 pages, La Jolla, CA, January 2008.