Random Linear Packet Coding: Joint Power and Rate Control

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***Abstract*—Random linear packet coding is considered for channels that experience fading and have long propagation delay, such as the underwater acoustic channels. Previously, we employed power control (adjusting the transmission power according to the channel gain) and rate control (adjusting the number of coded packets according to the channel gain) to counteract the effects of fading. For such policies, it was shown that there exists an optimal number of coded packets (when employing power control) or optimal transmission power (when employing rate control) for which the energy required per bit of information transmitted is minimized. In this paper, we present a strategy to jointly optimize the transmission power and the number of coded packets. We present two optimization criteria, i.e., (a) minimizing the average energy per bit for transmission and (b) maximizing the throughput. Given the statistics of the channel, we compute the average energy per bit and the average throughput under each criterion, to make a choice based on the desired trade-off. In applications with limited power, minimizing the average energy per bit will be the guiding principle, while in time-critical applications maximizing the average throughput will dictate the choice.**

I. Introduction

Underwater sensor networks play an important role in critical applications such as climate monitoring, area surveillance, ocean monitoring, off-shore oil and gas industry, etc. Electromagnetic waves do not propagate over long distances underwater, and hence acoustics signals are used for long-distance communication in underwater networks. However, acoustic communication poses a number of challenges including long propagation delays and Doppler distortion which arise due to the low speed of sound propagation in water. These effects, combined with the half-duplex nature of acoustic modems have a profound effect on the network performance.

Traditional automatic repeat request (ARQ) techniques rely on timely feedbacks from the receiver for optimal operation. Due to the half-duplex nature and long propagation delays, however, feedback become expensive in underwater networks. To counter this problem, we explore the use of random linear packet coding in combination with power and rate control for fading channels.

Random linear packet coding has been previously explored for underwater acoustic networks [1–5]. In this technique, the transmitter encodes a block of information-bearing packets into a larger set of coded packets to be transmitted, thereby increasing the chances of successful decoding at the receiver. In [1], rateless coding has been considered for reliable file transfer in underwater acoustic networks. Using a feedback link, the transmitter is informed when to stop sending the coded packets. Because the feedback is used less often than in a traditional ARQ scheme, the overall system efficiency is shown to improve. In [2], the authors investigate optimal broadcasting policies for underwater networks based on random linear packet coding, and show performance improvements over traditional ARQ techniques. In [3], optimal schedules are investigated that minimize the average time (or energy) needed to complete the transmission of a block of packets. Random linear packet coding in the absence of a feedback link has been investigated in [4]. The number of coded packets to transmit is determined so as to satisfy a pre-defined success rate. It is shown that when the packet error rate is reasonably low, it suffices to transmit only a few extra packet to ensure decoding with a high probability.

Random linear packet coding for fading channels was considered in [5]. In that work, we provided a framework to couple adaptive power and rate control with random linear packet coding. Under the assumption of a block-fading channel (in which the channel gain remains stable over a block of packets), we showed how the power (or rate) can be adjusted to maintain a pre-defined success rate at the receiver. With adaptive power control, we found that there is an optimal number of coded packets that can be transmitted so as to ensure that the average energy per bit is minimized. Similarly, there is an optimal transmit power that ensures minimum energy per bit while employing adaptive rate control.

Joint power and rate control has been extensively studied in the general wireless literature; however, not much has been done in the context of underwater acoustic links. In [6], the authors use a block-fading channel model, and define utility and cost functions to specify an optimization problem that aims at maximizing the rate while minimizing the power consumption. In [7], the authors consider a linear network and investigate the power consumption and bandwidth usage for information exchange between two terminal nodes.

In this paper, we extend the work of [5] to jointly optimize the power and the rate. We consider a block-fading channel model, in which the channel gain is decomposed into two parts: the large-scale, slowly-varying part which admits feedback, and the small-scale, faster-varying part which does not admit feedback and determines the (conditional) bit error rate performance. We rely on the feedback to convey the large-scale fading information from the receiver to the transmitter, and adjust the power and the rate (the number of coded packets to send) in accordance with the channel state. The adjustment is performed under an outage requirement imposed on the probability of successful decoding. We define two optimization criteria to jointly adapt the power and the rate. Under the first criterion, we aim to minimize the average energy per bit of information transmitted, while under the second criterion, we aim to maximize the average throughput between the transmitter and receiver.

The paper is organized as follows. In Sec.II, we present the system model. In Sec.III we specify the outage criterion and the resulting transmission method. In Sec.IV, we present the two optimization criteria for the joint power and rate control. Numerical results are presented in Sec.V, and conclusions are summarized in Sec.VI.

II. System Model

On a time-varying channel with a large-scale channel gain G and transmit power P_T, the averaged received power is given by $P_R = GP_T$. We assume a block-fading model, in which the channel gain remains constant within a block of packets, and may change from one block to the next. The transmitter encodes blocks of M original packets into $N > M$ coded packets and transmits them over the channel.

The signal-to-noise (SNR) ratio in each block is thus given by $\gamma = GP_T/P_N$, where P_N denotes the noise power. For a chosen modulation/coding method, the bit error rate (BER) of this channel is given by a functional relationship $P_e(\gamma)$, and the corresponding packet error rate is $P_E(\gamma) = 1 - (1 - P_e(\gamma))^{N_b}$, where N_b represents the number of information bits in a packet. Since the channel gain G is randomly varying, so is the packet error rate P_E.

The success rate, i.e., the probability that at least M out of the N coded packets are received correctly is given by

$$P_s = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma))^m P_E^{N-m}(\gamma) \tag{1}$$

We denote the pre-defined success rate that needs to be maintained at the receiver by P_s^*. The outage probability is now defined as the probability that the success rate falls below the pre-defined reliability, i.e.,

$$P_{out} = P\{P_s < P_s^*\} \tag{2}$$

For a given M and N, the outage probability can be represented in terms of the signal-to-noise ratio as

$$P_{out} = P\{\gamma < \gamma *\} \tag{3}$$

which, for a given power P_T, reduces to

$$P_{out} = P\{G < G_{out}\} \tag{4}$$

As shown in [8], we assume that the large-scale channel gain G is a log-normally distributed random variable, i.e., $g = 10\log_{10} G \sim \mathcal{N}(\bar{g}, \sigma^2)$. For a log-normally distributed random variable, the outage probability is given as

$$P_{out} = Q(\frac{\bar{g} - 10\log_{10} G_{out}}{\sigma}) \tag{5}$$

where $Q(.)$ denotes the Q-function, $Q(x) = 1/2\text{erfc}(x/\sqrt{2})$. The outage gain is thus given by

$$G_{out} = 10^{\bar{g} - \sigma Q^{-1}(P_{out})} \tag{6}$$

III. Outage Criterion

Power and rate control strategies are based on the information that the transmitter has about the channel. In each cycle, the receiver feeds the value of the channel gain G and the noise power P_N back to the transmitter. The transmitter thus observes the channel gain, and adjusts the power/rate accordingly.

To ensure that a given outage requirement is met, we focus on an on-off adjustment method. In this method, the transmitter compares the channel gain with the outage threshold (6) and proceeds as follows: If $G \geq G_{out}$, it chooses the transmit power and the rate (P_T, N) that ensure $P_s = P_s^*$; otherwise, it shuts off. Hence, whenever the transmitter is on, the success rate equals P_s^*, and that occurs with probability $1 - P_{out}$.

In practical systems, additional constraints are imposed by the limits on the maximal transmit power and the duration of a cycle. Specifically, the transmit power cannot exceed a certain maximum $P_{T,max}$, while the cycle duration has to comply with the assumption of block-fading. If we denote by T_{max} the time over which the large-scale gain G can be considered constant, then the number of coded packets that fill this time cannot exceed a certain value N_{max}. Note that this value also depends on the packet duration $T_p = N_b/R_b$, i.e., the number of bits per packet and the bit rate R_b. Without loss of generality, we set $N_{max} = T_{max}/T_p$.

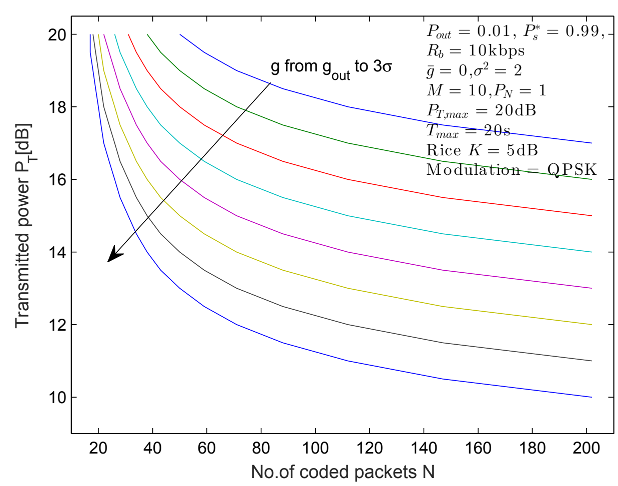

Figure 1. (P_T, N) pairs that satisfy the condition $P_s = P_s^*$.

IV. Joint Power and Rate Control

From Fig.1, two special cases can be observed. In the first case, for a fixed N (vertical cut across the curves), we can adjust the transmit power P_T in accordance with the observed channel gain G. In the second case, for a fixed P_T (horizontal cut across the curves), we can adjust the number of coded packets in accordance with the observed channel gain G so that $P_s = P_s^*$.

A third approach is also possible, in which the power and the rate are adjusted jointly. The way in which this adjustment is made depends on the optimization criterion. We consider two criteria: minimizing the average energy per bit, and maximizing the average throughput. Each criterion results in a control policy defined as $P_T(G), N(G)$.

A. Average energy per bit

Under this optimization criterion, we aim to minimize the average energy per bit of information used for transmission. The average energy per bit is given by

$$\bar{E}_b = \frac{1}{P_s^* M R_b} \int_{G_{out}}^{\infty} N(G) P_T(G) p_G(G) dG \qquad (7)$$

where R_b represents the bit rate of the channel , and $p_G(\cdot)$ is the probability density function (pdf) of the gain G. Because the pdf is non-negative, $\bar{E}_b$ will be minimized when P_T and N are chosen such that their product is minimized for a given G. The resulting policy $P_T(G), N(G)$ is illustrated by the solid curves in Fig.2.

B. Average throughput

Under this optimization criterion, we aim to maximize the average throughput. The average throughput is given by

$$\bar{R} = P_s^* M R_b \int_{Gout}^{\infty} \frac{1}{N(G)} p_G(G) dG \qquad (8)$$

In this scheme, we simply aim to reduce the number of coded packets that need to be transmitted. The resulting policy $P_T(G), N(G)$ is illustrated by the dashed curves in Fig.2.. Reducing the number of coded packets implies an increase in power if the success rate is to be kept at a given value P_s^*. Note that the minimum value of N is thus determined by the maximum available power $P_{T,max}$. In contrast, minimizing the average energy per bit leans towards lower values of the power and larger values of N, which are capped by N_{max}, as dictated by the large-scale coherence time.

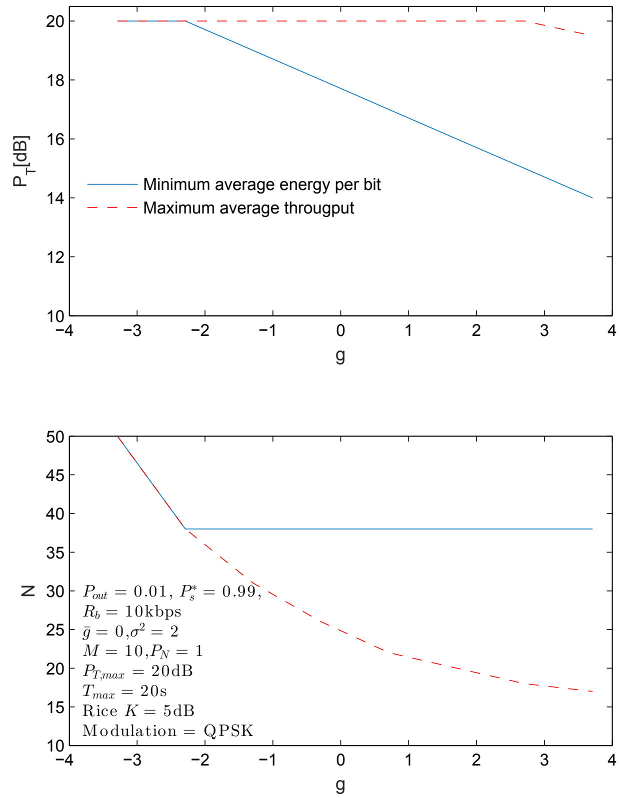

Figure 2. P_T and N choices under the two optimization criteria, i.e., minimum average energy per bit and maximum average throughput. When $G < G_{out}$, the transmission is shut off. For a practical implementation, this example shows two main regions of operation under minimum energy criterion. In the low gain regions, the power is constant and the number of coded packets is adjusted, while in the higher gain regions, the number of coded packets remains constant and the power is adjusted. Under the maximum throughput criterion, the power is kept constant while the number of coded packets is adjusted.

V. Numerical Evaluation

Given the distribution of the gain, it is possible to evaluate the average energy per bit and the average throughput for each policy, and hence a quantitative comparison is possible enabling one to choose the transmit power and the rate based on a desired trade-off between the energy and the throughput.

To illustrate the system performance, we use the log-normal fading model for the large-scale channel gain. We assume QPSK modulation scheme and a Ricean model for the small-scale fading. For purposes of illustration, we use the BER characteristic of a frequency non-selective channel, noting that an actual acoustic channel likely does not conform to this model, and may include additional multipath diversity. Table.I lists the system parameters that we use.

Parameter	Value
P_{out}	0.01
P_s^*	0.099
R_b	10 kbps
T_{coh}	300s
$P_{T_{max}}$	20 dB
P_N	1
K	5 dB

Table I
SYSTEM PARAMETERS.

Fig.3 and Fig.4 show the average energy per bit and the average throughput for the two optimization criteria as functions of the block size M. It can be observed that maximizing throughput can be achieved at the expense of higher average energy per bit. One can choose which optimization criterion to use based on the specific system requirements. For instance, in short-term, time-critical missions, such as high-rate image transmission from an AUV, one might favor throughput over energy, while in long-term sensor deployments for low-content, infrequent environmental data telemetry, the opposite may be the preferred choice.

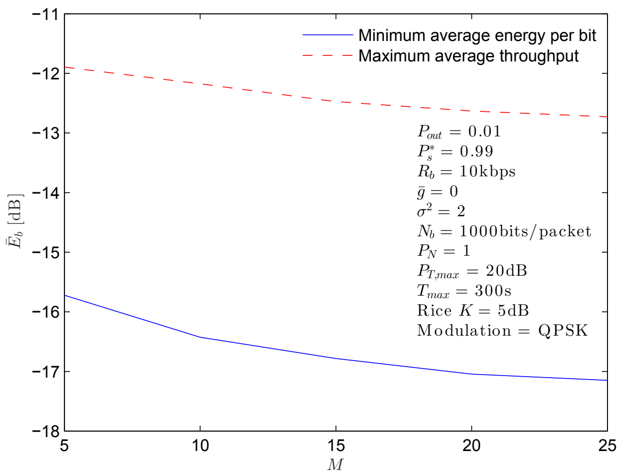

Figure 3. Average energy per bit as a function of the block size M.

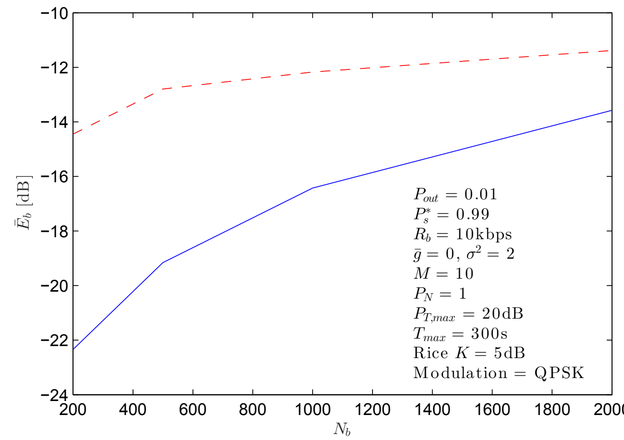

Figure 5. Average energy per bit as a function of the packet length N_b.

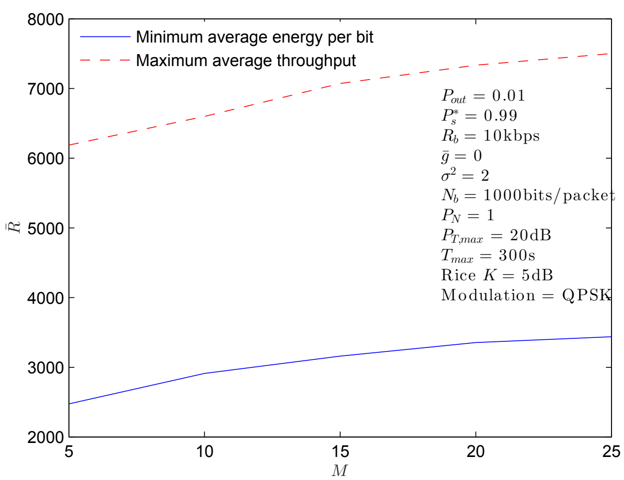

Figure 4. Average throughput as a function of the block size M.

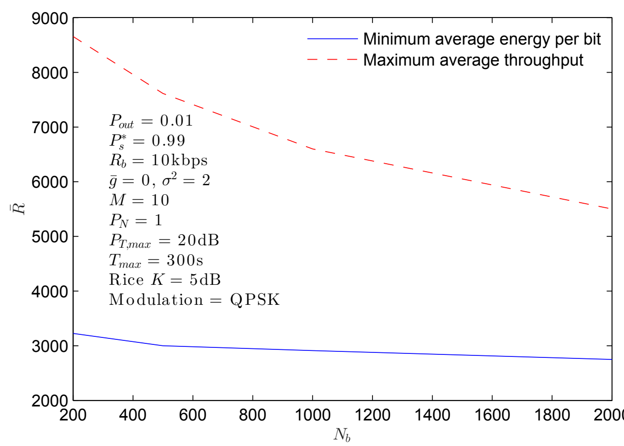

Figure 6. Average throughput as a function of packet length N_b.

Fig.5 and Fig.6 show the average energy per bit and the average throughput for the two schemes as a function of the packet size N_b. It can be observed that an increase in the packet size directly influences the packet error rate and hence leads to a higher average energy per bit.

VI. Conclusion

Random linear packet coding provides a simple, yet efficient alternative to traditional ARQ techniques which suffer on channels with high latency. We combined random linear packet coding with adaptive power and rate control in a system design based on an outage requirement. We proposed a joint power and rate control policy for a block-fading channel where the large-scale channel gain does not change over a certain period of time. For each such period, the receiver feeds the channel information back to the transmitter, which then adjusts its power and the code rate based on one of two criteria: minimizing the average energy per bit of information, or maximizing the average throughput. Given the distribution of the channel gain, which we assumed to be log-normal, the average energy per bit and the average throughput were computed.

The choice of a suitable optimization criterion depends on the particular application. In applications that involve long-term deployments, limited power is available and minimizing the average energy per bit is important for prolonging the system's lifetime. In time-critical, short-term applications such as communication to and from an AUV, throughput might take precedence over energy.

Future work will focus on relaxing the block-fading model assumption and on including acoustics-specific small-scale BER models. In addition, the work will be extended to multicast networks.

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