# Random Linear Packet Coding for Broadcast Networks

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Abstract-Random linear packet coding is considered for efficient broadcasting in networks with large propagation delays, such as underwater acoustic networks. To additionally overcome the effects of fading, we combine packet coding with adaptive power control, whereby the transmitter adjusts its power upon receiving feedback from the receiver on the current state of the channel (locally-averaged, large-scale gain). We investigate two power adjustment rules: the worst link rule and the average link rule. In the first case, the transmit power is adjusted in accordance with the link that has the lowest channel gain, while in the second case, the power is adjusted in accordance with the average of the gains on all links. System performance is evaluated based on the average energy per bit of successfully transmitted bit of information, using (i) simulated channels, (ii) experimentally recorded gain values from the MISSION 2012 experiment, and (iii) actual network deployment from the MISSION 2013 experiment conducted off the coast of Singapore. Results indicate energy savings on the order of several dB compared to systems that do not use power control.

### I. INTRODUCTION

Underwater networks have garnered a lot of interest in recent years, notably for applications such as environmental, industrial, and coastal monitoring. Network efficiency, however, remains limited by the channel latency and the halfduplex nature of existing modems, necessitating system design beyond traditional automatic repeat request (ARQ) techniques. To address this issue, we investigate the use of random linear packet coding for efficient broadcasting in underwater acoustic networks.

Random linear packet coding is based on the simple fact that if one transmits not only the information-bearing data packets, but a larger set of their coded combinations, the chances of successful reception will be increased without having to wait for individual packet acknowledgments. In the context of acoustic networks, random linear packet coding has been investigated in [1–6]. In [1], the authors investigate the use of rateless codes for reliable file transfer in underwater acoustic networks. A feedback link is used to inform the transmitter when to stop the coded packet steam, and it is shown that the efficiency improves since feedback is used less frequently than with individual packet acknowledgments. Half-duplex operation, however, mandates that a feedback schedule be determined a-priori. Optimal schedules which minimize the average packet transmission time (or energy) are developed in [2]. In [3], the use of random linear packet coding without

feedback is described. The number of coded packets is now determined so as to satisfy a pre-defined success rate.

Random linear packet coding is particularly meaningful in broadcast situations, where packet errors are uncorrelated between different receivers. Optimal broadcasting in underwater networks was addressed in [4]. There, rateless codes are used to transmit a block of packets to all the nodes in a network, and it is shown that efficiency increases in comparison with traditional ARQ techniques. Broadcasting in time division duplexing channels was addressed in [5], where it is shown that rateless packet coding outperforms other scheduling policies.

All of the previously mentioned work assumes a fixed, nonfading channel. In [7], rateless codes are investigated for use over fading channels with delay constraints. In [8], the use of rateless codes in a relay network is discussed. Under the assumption of a flat fading Rayleigh channel and a delay constraint, the authors develop strategies for finding the best rate that ensures an acceptable probability of outage. In our previous work [6], a framework was developed for coupling random linear packet coding with adaptive power control over fading channels.

In this paper, we extend the work of [6] to a broadcast network scenario. Packets are transmitted in blocks, and we assume that the large-scale channel gain remains constant over a block of packets (block-fading model). The channel gain information is available at the transmitter through the feedback, and the transmitter uses it to adjust the power such that a pre-defined reliability (success rate) is achieved. We investigate two power adjustment rules: the worst link rule, where the transmit power is adjusted in accordance with the link that has the worst (lowest) channel gain, and the average link rule, where the transmit power is adjusted in accordance with the average of the gains on all links.

We demonstrate the system performance through simulation, emulation, and an at-sea network deployment. Simulations are conducted using the the UNET underwater acoustic network simulator [9]. System emulation is performed using experimentally recorded gain values from the MISSION 2012 experiment, conducted off the coast of Singapore. Finally, we present results from the MISSION 2013 experiment, a realtime at-sea network deployment that took place in November 2013 in the same location.

The rest of the paper is organized as follows. In Sec.II, the

system model is described. In Sec.III, power control strategies are outlined. Simulation and experimental results are presented in Sec.IV. Conclusions are summarized in Sec.V.

#### II. SYSTEM MODEL

We assume a network deployment with a *leader* node that wishes to broadcast information packets to all the *receiver* nodes. In each broadcast cycle, the leader buffers M original packets and encodes them into N > M coded packets. It then proceeds to transmit the N packets over the acoustic link. The number of coded packets N is fixed, and is chosen such that the average energy per bit of information is minimized for the average link (see [6]).

We denote the channel gain between the leader and  $i^{th}$  node by  $G_i$ . The received power at the  $i^{th}$  node is thus  $P_{R_i} = G_i P_T$  and the resulting SNR is  $\gamma_i = P_{R_i}/P_N$ , where  $P_N$ is the noise power. The packet error rate on the  $i^{th}$  link is denoted by  $P_E(\gamma_i)$ . Once the receiver has obtained sufficiently many coded packets for successful decoding, it will inform the transmitter about the channel gain and the noise power.

The leader can now adjust the power such that a pre-defined reliability is met. The reliability, i.e. the probability of success, is the probability that M or more out of the N packets are received correctly:

$$P_s = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma))^m P_E^{N-m}(\gamma)$$
(1)

For a pre-specified success rate  $P_s^*$ , the outage probability is defined as

$$P_{out} = P\{P_s < P_s^*\} \tag{2}$$

We can now adjust the transmit power  $P_T$  so that the reliability  $P_s$  is kept constant at the desired level  $P_s^*$ . We use two rules for adjusting the transmit power  $P_T$  as discussed next.

#### **III. POWER CONTROL STRATEGIES**

In terms of the SNR, the probability of outage is expressed

$$P_{out} = P\{\gamma < \gamma^*(N)\} = P\{G < G_{out}(P_T, N)\}$$
(3)

where  $\gamma^*(N)$  is the SNR corresponding to a given  $P_s^*$ . If the distribution of the gains is known,  $\gamma^*(N)$  can be computed. Following the results of [10], we assume that the large-scale channel gain is a log-normally distributed random variable,  $g = 10 \log_{10} G \sim \mathcal{N}(\bar{g}, \sigma^2)$ . The probability of outage is then given by

$$P_{out} = Q(\frac{\bar{g} - 10\log G_{out}}{\sigma}) \tag{4}$$

and the outage gain is thus given by

$$G_{out} = 10^{\bar{g} - \sigma Q^{-1}(Pout)} \tag{5}$$

where Q(x) denotes the Q-function. Note that the average gain (and variance) of different links in a network can be different; hence, each link may have a different outage threshold.

For a fixed N, in the absence of fading, i.e. when G = 1, the power required to achieve the reliability condition is given by

$$P_{T_0} = \gamma^*(N) P_N \tag{6}$$

In the presence of fading, the transmitter must adapt its power for each block of packets in accordance with the channel gain. Below, we specify two rules for transmit power adaptation.

#### A. Worst Link Rule

According to this rule, the transmitter adapts its power in accordance with the link that has the worst (lowest) channel gain. After receiving feedback from all the receivers, the transmitter computes

$$G_{min} = \min_{i:G_i \ge G_{out,i}} \{G_i\},\tag{7}$$

and

$$G_{out,min} = \min_{i:G_i \ge G_{out,i}} \{G_{out,i}\}$$
(8)

and adjusts its power as

$$P_{T_W} = \begin{cases} P_{T_0}/G_{min}, & G_{min} \ge G_{out,min} \\ 0, & \text{otherwise} \end{cases}$$
(9)

The average energy per bit consumed under this adaptation rule is given by

$$\bar{E}_{b_W} = \frac{P_{T_W}N}{MR_b} \tag{10}$$

where  $R_b$  is the bit rate.

## B. Average link rule

In this case, the leader node computes the mean of the channel gains as

$$\overline{G} = \frac{1}{D-1} \sum_{i:G_i \ge Gout, i} G_i \tag{11}$$

where D is the total number of nodes, including the leader. The transmit power is now adjusted as

$$P_{T_M} = \begin{cases} P_{T_0}/\overline{G}, & \overline{G} \ge G_{out,min} \\ 0, & \text{otherwise} \end{cases}$$
(12)

The average energy per successfully transmitted bit of information is given by

$$\bar{E}_{b_M} = \frac{\bar{P}_{T_M} N}{M R_b} \tag{13}$$

#### **IV. RESULTS**

The performance of the two power control algorithms are presented here using simulation and experimental data. Simulation is performed using the UNET underwater acoustic network simulator [9]. Experimental results come from the MISSION 2012 and MISSION 2013 experiments, which are described in Secs.IV-B and IV-C. From the MISSION 2012 experiment, we use the recorded channel gain values to emulate the system performance, while the MISSION 2013 experiment involves a real-time network deployment at sea.

#### A. Simulation results

For simulation, we use a network of six nodes, placed at equal distance away from the leader. The large-scale channel gain is modeled by a log-normal distribution, with same parameters for all the links. To begin, we set the parameters of the log-normal distribution to  $(\bar{g}, \sigma^2) = (-30, 5)$ , and look at a design based on  $P_{out} = 0.01$  and  $P_s^* = 0.99$ . Assuming QPSK modulation and a Ricean model for the small-scale fading, the bit error rate  $P_e$  is given by [11]

$$P_e(\gamma) = \frac{1+k}{2+2k+\gamma} \exp(-\frac{k\gamma}{2+2k+\gamma})$$
(14)

where k is the Ricean k-factor. The above expression provides the bit error rate (BER) for a frequency non-selective channel. In the presence of frequency selectivity, additional multipath diversity gain is available through equalization. For the sake of generality, we omit this gain (which depends on a particular multipath intensity profile [12]) and use the expression (14) as it suffices to provide insight into the general BER trend.

Fig.1 shows the average energy savings per bit, for different block sizes M, when employing the worst link rule and the average link rule. The average link rule is an opportunistic method where the pre-specified reliability is not guaranteed to every node. Hence, the average link rule will use less power when compared to the worst link rule and this is evident from Fig.1.



Figure 1. Simulation results for the two power control algorithms using the log-normal fading model.

The fading model parameters have a pronounced effect on the performance. Fig.2 shows the results of varying the mean  $\bar{g}$ for the two power control algorithms. As expected, increasing the mean implies that less power is required for transmission and consequently less energy per bit. As before, the average link rule provides higher energy savings.



Figure 2. Simulation results for varying mean of the channel gain  $\bar{g}$ . Increasing the mean implies a favorable channel; hence, less energy is required to achieve a given reliability.

Fig.3 shows the results of varying the variance  $\sigma^2$  for the two power control algorithms. As the variance increases, additional power is required to maintain reliability, and this in turn increases the average energy per bit.



Figure 3. Simulation results for varying the variance of the channel gain  $\sigma^2$ .

The above results clearly show that random linear packet coding for information broadcast provides energy savings on the order of a few dB. These results are further verified using experimental data.

#### B. MISSION 2012

The MISSION 2012 experiment included multiple network deployments near Palau Hantu, off the coast of Singapore in October 2012. The network consisted of Unet-PANDA nodes, each equipped with an anchor, an underwater modem, a battery pack, and an acoustic release buoy [13]. The modem assembly rested a few meters above the anchor on the ocean floor. Fig.4 shows a typical set-up of the Unet-PANDA node. Each Unet-PANDA is equipped with an ARL UNET-2 modem, which operates in the 18 - 36 kHz band and can communicate up to a range of 2.5 km, using different modulation schemes.



Figure 4. Unet-PANDA typical set-up.

During the MISSION 2012 experiment, the network deployment consisted of five nodes including four Unet-PANDA nodes and one surface modem that was deployed from a barge. We use the surface node as the leader and the other Unet-PANDA nodes as receivers. Fig.5 shows the MISSION 2012 deployment.



Figure 5. MISSION 2012 deployment. Each Unet-PANDA node is marked by P and the barge surface modem is marked by B.

Fig.6 shows the histogram of the channel gains recorded between nodes 21 and 27. We can clearly see a good fit between the normal distribution and the data. Fig.7 shows the corresponding cumulative distribution functions.

The experimentally recorded gains were used to emulate the performance of power control algorithms. Results are shown in Fig.8. As before, energy savings on the order of a few dB are available by employing the adaptive power control algorithms.



Figure 6. Histogram of gains recorded during the MISSION 2012 experiment on the channel between nodes 21 and 27.



Figure 7. Cumulative distribution of the recorded gain g between nodes 21 and 27. The 95% confidence intervals and the normal distribution fit is also plotted.

#### C. MISSION 2013

The MISSION 2013 experiment was conducted in November 2013, also near Palau Hantu off the coast of Singapore. The hardware used in the MISSION 2013 experiment was the same as the MISSION 2012 experiment although the network deployment was slightly different. Seven Unet-PANDA nodes were deployed in shallow water with an average depth of about 15 m. The leader node was deployed from an anchored barge station in the middle of the network. Fig.9 shows the network deployment for the MISSION 2013 experiment. The nodes were submerged and the only possible communication was via acoustics.



Figure 8. Experimental results using the channel gains recorded during the MISSION 2012 experiment.



Figure 9. MISSION 2013 network deployment.

The received signal power was captured in each block using a probe consisting of a 500 ms long m-sequence transmitted before the packets. Each node also computed the noise power by recording a period of silence before the packet transmissions. Once sufficiently many packets were received for successful decoding, each receiver sent the feedback packet containing the received signal power and noise power. For the real-time implementation, a block size of M = 10 packets was chosen. Using the statistics of the channel available from the previous experiment, it was determined that N = 16 packets used the least average energy per bit.

The actual power used in each block is shown in Fig. 10. We can see that energy savings of about 6 dB are achieved by employing adaptive power control using the average link rule while maintaining a reliability of  $P_s^* = 0.99$ .

Due to the limited availability of the network resources for testing, we were only able to test the performance of the algorithm online using the average link rule. However, using the experimentally recorded gain values, we emulated the system performance using the worst link rule. Combined



Figure 10. MISSION 2013 results. Real-time run of the power control algorithm using the average link rule.

results are shown in Fig.11.



Figure 11. Results of the MISSION 2013 experiment.

#### V. CONCLUSION

We addressed acoustic network broadcast within the framework of random linear packet coding. For a block fading channel, where the large-scale channel gain remains constant over a block of packets, we proposed adaptive power control that adjust the transmit power in accordance with the channel gain in order to minimize the average energy required per successfully transmitted bit of information. Two adaptation rules were investigated: the worst link rule, where the transmit power is adjusted in accordance with the link that has the lowest channel gain, and the average link rule, where the transmit power is adjusted in accordance with the average of the channel gains reported on all the links. Energy savings resulting from packet coding and power control were quantified using simulation and experimental data. Good agreement was observed between the experimental results and the synthetic results obtained using log-normal distribution for the large-scale channel gain. Results from two experiments show that energy savings on the order of several dB are available from the proposed power control strategies. For the future work, we will concentrate on joint power and rate control strategies to maximize energy savings.

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