

# The Acoustic Channel and Delay: A Tale of Capacity and Loss

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**Abstract**—We analyze the capacity of single-input-multi-output (SIMO) acoustic channels with long propagation delays and long-term frequency-dependent properties that are found in acoustic channels. The focus of this paper is on the effects of feedback delay on the achievable rate of the acoustic channel. Four power allocation policies are analyzed within a framework of orthogonal frequency division multiplexing (OFDM): water-filling, statistical water-filling (water-filling based on long-term channel statistics), frequency band selection (uniform power allocated to all carriers within a range of frequencies), and uniform power allocation across all carriers irrespective of the channel (“all-on”). The first three methods show very little difference in rate for average to high SNRs. However, the frequency-band-selection method is robust against feedback delay, requires little overhead and has low complexity, thus making it the method of choice. In mobile systems where the distance between the transmitter and receiver vary significantly, adaptive frequency band selection can deliver a significant gain compared to the “all-on” policy, as it picks the band that is statistically optimal for the given distance. Finally, we show that although water-filling can increase the capacity significantly in SNR-starved systems with perfect channel knowledge at the transmitter, the gain turns into a loss with the long feedback delays associated with underwater acoustic channels.

**Index Terms**—Underwater acoustic communications, channel capacity, information rate, Rician fading, water-filling, power allocation, OFDM.

## I. INTRODUCTION

While much attention has been paid to the capacity of wireless channels (see e.g. [1], [2]), there has been limited research on the capacity of the acoustic channel. Recently, Radošević et al. [3], [4], and Socheleau et al. [5] studied the acoustic channel focusing on a Rician channel model that was supported by experimental measurements. However, their study did not take into account the effect of propagation delays and the resulting channel estimation errors at the transmitter that result from them. In [6], [7], we investigated the affect of channel estimation and the required pilots on the rate of the acoustic channel.

In this paper, we focus on calculating the average achievable rate for a class of underwater acoustic channels recently been analyzed in [6], where the short term channel variations where considered. The results of [6] shows that there is very little difference between water-filling and uniform power distribution when water-filling is applied based on short-term channel variations. Here in contrast, we include acoustic-specific propagation with frequency-dependent attenuation and colored noise in the channel model to show the benefit

of spectrum shaping given the frequency dependent channel statistics.

To analyze the rate of the underwater channel, we assume an Orthogonal Frequency Division Multiplexed (OFDM) modulation. By comparing four power-allocation strategies: water-filling, statistical water-filling (water-filling based on frequency dependent channel statistics), frequency band selection (uniform power across the frequency band utilized by statistical water-filling), and all-on (uniform power across the spectrum regardless of the channel statistics), we show that statistical water-filling is the most robust method in the presence of long propagation delays.

Our analysis also extends to single-input-multi-output (SIMO) channels where the channel is estimated at the receiver, the information is sent with delay to the transmitter. This analysis is based both on the experimental data collected during the 2010 mobile acoustic communications experiment (MACE’10), as well as a simulation model introduced in [8], where each propagation path is a non-zero-mean, first-order auto-regressive complex-Gaussian random process.

The paper is organized as follows. In Sec. II we introduce the acoustic channel model. This model includes short-term channel variations as well as long-term frequency-dependent attenuation and colored noise. In Sec. III we define the achievable rate and introduce the power allocation strategies. In Sec. III-A we discuss feedback strategies and the impact of delay. Sec. IV contains the numerical results on the achievable rate based on simulation and experimental data. Conclusions are summarized in Sec. V.

## II. CHANNEL MODEL

We consider a SIMO channel with  $M$  receiver elements which can be described by a set of instantaneous transfer functions  $H^1(f), \dots, H^M(f)$ . Using OFDM modulation, we can describe the channel using the transfer functions at the subcarrier frequencies  $f_k = f_0 + k\Delta f$ ,  $k = 0, \dots, K-1$ , where  $\Delta f = 1/T$  and  $T$  is the length of the OFDM inverse Fast Fourier Transform (IFFT) period. We denote by  $H_k^m$ , the frequency response at center frequency  $f_k$ ,  $H^m(f_k)$ . The received signal at the  $k$ -th carrier can now be modeled in vector form as

$$\mathbf{y}_k = \sqrt{P_k} \mathbf{H}_k d_k + \mathbf{z}_k \quad (1)$$

where  $\mathbf{H}_k = [H_k^1, \dots, H_k^M]^T$  is the vector of channel transfer functions,  $P_k$  is the power allocated to the  $k$ -th carrier,  $d_k$

is the information signal transmitted on this carrier which is assumed to have unit variance, and  $\mathbf{z}_k$  is zero-mean, complex Gaussian noise of variance  $\sigma_{z_k}^2$  for each receiver element. The total transmit power over the bandwidth  $B = K\Delta f$  is  $P_{tot}$ .

We employ a channel model where each receiver element has no more than  $P$  propagation paths. The  $p$ -th path of the  $m$ -th receiving element during the  $n$ -th block is denoted by  $h_p^m(n)$  with a delay  $\tau_p^m(n)$ . Assuming that the channel variations during one OFDM block are negligible, the transfer function for  $k$ -th carrier of the  $m$ -th receiver element is

$$H_k^m(n) = \frac{1}{\sqrt{A_k}} \sum_p h_p^m(n) e^{-j2\pi f_k \tau_p(n)} \quad (2)$$

where  $A_k$  is introduced to account for the frequency dependent attenuation. Note that there are two sources for frequency selectivity: multipath spread and frequency-dependent channel statistics ( $A_k$  and  $\sigma_{z_k}^2$ ). However, frequency-dependent statistics vary slowly over time and therefore can be fed back to the transmitter for spectrum shaping.

The path gains within each receiver element are modeled as independent, first-order auto-regressive processes with:

$$[h_p^m(n+1) - \bar{h}_p^m] = \rho_{p,m}[h_p^m(n) - \bar{h}_p^m] + \sqrt{(1 - \rho_{p,m}^2)}\chi_p^m \quad (3)$$

where  $\bar{h}_p^m = E\{h_p^m(n)\}$  denotes the mean value of the gain,  $\sigma_{p,m}^2 = E\{|h_p^m(n) - \bar{h}_p^m|^2\}$  represents the variance from the mean, and  $z_p^m \sim \mathcal{CN}(0, 1)$  is the process noise which is uncorrelated with  $h_p^m(n-1)$  as well as across  $m$ ,  $p$  and  $n$ . This results in a Rician fading process for which we define the average Rician  $K$ -factor as  $\bar{K} = \sum_p (\bar{h}_p^m)^2 / \sum_p \sigma_{p,m}^2$ . We also assume a normalization of path gains  $h_p^m$  such that

$$\sum_p (|\bar{h}_p^m|^2 + \sigma_{p,m}^2) = 1, \quad m = 1, \dots, M \quad (4)$$

which implies the average SNR of

$$SNR_k = P_k \frac{E\{H_k^m\}}{\sigma_{z_k}^2} = \frac{P_k}{A_k \sigma_{z_k}^2} \quad (5)$$

for each receiver element. Because the attenuation  $A_k$  and the noise  $\sigma_{z_k}^2$  lower the effective SNR of the subcarrier, the term  $A_k \sigma_{z_k}^2$  is a good metric for quality of each subcarrier. Fig. 1 illustrates this quantity, obtained for spherical spreading.

The path delays are modeled as

$$\tau_p^m(n) = \tau_p^m(n-1) - a_p^m \cdot (T + T_g) \quad (6)$$

where  $a_p^m$  is the Doppler factor that captures motion-induced time scaling on the  $p$ -th path to the  $m$ -th receiver element. We assume that the receiver has no *a priori* knowledge of the Doppler.

### III. ACHIEVABLE RATE AND POWER ALLOCATION

To determine the effect of propagation delay on the rate, we assume that the receiver knows the channel perfectly, but this information is sent to the transmitter with a delay. In such systems, the instantaneous achievable rate follows from the

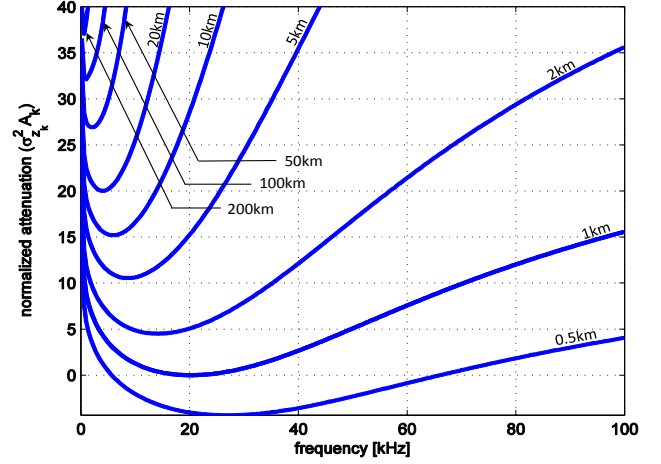


Fig. 1. This plot displays the SNR loss factors:  $\sigma_{z_k}^2 A_k$  vs. frequency. Normalization is performed such that the 0 dB level corresponds to the minimum at 1 km. The noise profile is generated as in [9] with the parameters set to no wind ( $w = 0$ ) and moderate shipping activity ( $s = 0.5$ ). The spreading is assumed to be spherical.

Shannon capacity formula (see e.g. [10] for details),

$$R(n) = \frac{T}{T + T_g} \cdot \Delta f \sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{P_k \|\mathbf{H}_k(n)\|^2}{\sigma_{z_k}^2} \right) \quad (7)$$

where  $T_g$  is the guard interval and  $\|\cdot\|$  denotes  $L_2$  norm of a vector. Note that this rate results from maximum ratio combining at the receiver. The instantaneous rate is a random variable and therefore we use the notion of average rate,  $\bar{R} = E[R(n)]$ , taken over multiple channel realizations to measure the performance.

#### A. Outdated Feedback

Delayed feedback causes the transmitter's estimate (which is used to allocate the power) to be outdated even if the receiver feeds a perfect channel estimate back to the transmitter. Propagation delay plays a dominant role in the acoustic channel. In addition, the acoustic channels typically vary at a high rate, thus fundamentally limiting the amount of reliable channel information that the receiver can feedback to the transmitter.

Here, we investigate four power allocation policies:

1) *Imperfect water-filling*: Given perfect channel knowledge at the transmitter, the power allocation strategy that maximizes the rate is water-filling. When the channel information is available with a delay, however, the transmitter can only imitate water-filling, which we denote here as imperfect water-filling. This policy allocates power according to the water-filling rule, but the outdated channel information  $\mathbf{H}_k(n - D_t)$  replaces the current value,  $\mathbf{H}_k(n)$ :

$$P_k(n) = \begin{cases} \nu - \frac{\sigma_{z_k}^2}{\|\mathbf{H}_k(n - D_t)\|^2}, & \text{when } \|\mathbf{H}_k(n - D_t)\|^2 > \frac{\sigma_{z_k}^2}{\nu} \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

and the water level  $\nu$  is determined such that

$$\sum_{k=0}^{K-1} P_k = P_{tot} \quad (9)$$

2) “Statistical Water-filling”: Since the channel statistics, unlike multipath, change slowly over time, the transmitter can be informed about general channel statistical parameters, i.e. the value  $A_k \sigma_{z_k}^2$  for all carriers. Given this parameter, which we assume to be the same for all receiving elements, statistical water-filling is applied as follows:

$$P_k = \begin{cases} \nu - \frac{\sigma_{z_k}^2 A_k}{M}, & \text{when } \nu > \frac{\sigma_{z_k}^2 A_k}{M} \\ 0, & \text{otherwise} \end{cases}, \quad k = 0, \dots, K-1 \quad (10)$$

where the water level  $\nu$  is selected as before, according to (9). We assume maximum ratio combining is applied at the receiver, increasing the average SNR  $M$ -fold.

3) *Frequency band selection*: This strategy is a simplified version of Statistical water-filling and assigns uniform power to all carriers for which  $\nu > \frac{\sigma_{z_k}^2 A_k}{M}$ , with value of  $\nu$  as calculated from (9) and (10), while nothing is given to the rest. This method takes advantage of the frequency correlation of the attenuation factors  $A_k$  and  $\sigma_z^2$ , and therefore has the added benefit of greatly reducing the required feedback in the system. The receiver needs only to send the frequency boundaries of the chosen band. Because the effect of the frequency band attenuation is very similar to a band-pass filter, this method will in general cut off the lower and higher frequencies of the signal. It is an intelligent way to determine the effective bandwidth of the usable acoustic spectrum.

4) *All-on Uniform*: This strategy distributes the power uniformly across the available bandwidth. In this paper, we will use the frequency band between 10 kHz - 15 kHz for this strategy, which matches the frequency band used during the experiment.

#### IV. RESULTS

In this section we compare the four power allocation policies in presence of feedback delay. The results are obtained using simulation and experimental data. The experimental data were recorded during the 2010 mobile acoustic communication experiment (MACE’10) which was conducted in a 100 m deep, 3 km - 7 km long mobile channel, with 256 carriers occupying 10 kHz - 15 kHz acoustic band (see [11] for details of deployment). A total of 1664 OFDM blocks were transmitted over a period of 3.5 hours. The guard interval is  $T_g = 16$  ms and the block duration is  $T = 51$  ms for both the experimental data and simulation. The simulated channel follows the model of Sec. II, where the average path gains and path delays are selected according to the channel geometry that matches the experimental one. We select 50 different channel responses (which vary slightly in the placement of transmitter and receiver) and add random time-variation over the duration of 100 OFDM blocks (5000 blocks in total). To describe the variation, we use the notion of average Rician

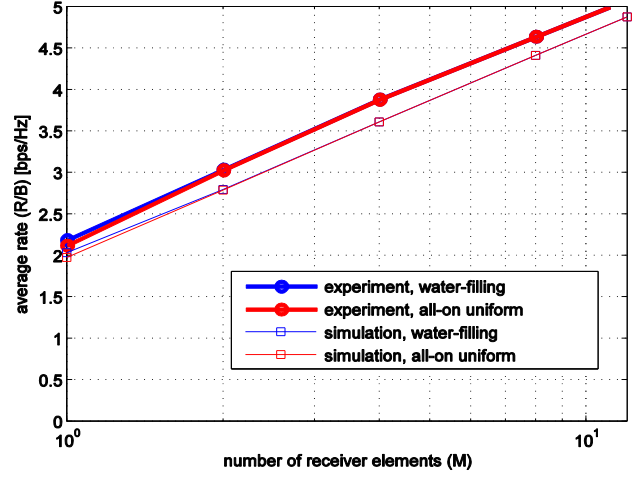


Fig. 2. Average rate calculated using experimental channel and simulation vs. number of receiving elements. The average SNR is 10 dB for each receiver element, and the transmitter and the receiver have perfect knowledge of the channel.

K-factor as introduced in [12],  $\bar{K}_m = \sum_p (\bar{h}_p^m)^2 / \sum_p \sigma_{p,m}^2$ , which is assumed to be the same for all receiver elements.<sup>1</sup>

First, we compare water-filling to the all-on uniform policy in terms of the average rate  $\bar{R}$  using a simplified model which assumes  $A_k = 1$  for all carriers and that the noise is white. In this case, rate can be expressed as  $R/B$  in bps/Hz. Note that we omit statistical water-filling and frequency band selection for this result as they will allocate power identical to the all-on uniform policy. Fig. 2 shows  $\bar{R}/B$  vs. number of receiving elements  $M$ , using both experimentally measured and synthetic channels. The average rate increases linearly with the logarithm of the number of receiver elements and water-filling is shown to provide negligible gain in rate, providing about 5 bps/Hz at the SNR of 10 dB when the signals from all 12 receivers are combined ( $M = 12$ ).

Figs. 3 - 5 demonstrate the effect of frequency-dependent attenuation and colored noise on the average rate. These figures are based on normalized transmit power, where the 0 dB level corresponds to the power required to sustain an average SNR of 0 dB over a bandwidth of 1 Hz at the statistically favorable frequency at a distance of 1 km. Frequency-dependent channel parameters are calculated as shown Fig. 1.

Figs. 3 and 4 compare four power allocation policies as a function of distance, assuming ideal channel knowledge both at the transmitter and the receiver. Water-filling provides the most significant gain at lower transmit powers (10 - 30 dB).

<sup>1</sup>The average multipath profile is characterized by the mean gain magnitudes, equal for all receiver elements,  $1\beta, 0.9\beta, 0.5\beta, 0.45\beta, 0.4\beta, 0.3\beta, 0.1\beta$  and nominal delays 0, 0.4, 1.7, 3, 5.5, 7.6, 11.5 ms. The corresponding variances are  $1\gamma, 0.2\gamma, 0.5\gamma, 0.15\gamma, 0.15\gamma, 0.07\gamma, 0.1\gamma$ , and the factors  $\beta$  and  $\gamma$  are selected such that the normalization  $\sum_p (\bar{h}_p^m)^2 + \sum_p \sigma_{p,m}^2 = 1, m = 1, \dots, M$  is satisfied and the desired average Rician K-factor,  $\bar{K} = \sum_p (\bar{h}_p^m)^2 / \sum_p \sigma_{p,m}^2$ , is achieved. Doppler scaling factors are generated independently for each path and receiver element as Gaussian distributed with zero mean and standard deviation  $\sigma_a$  which varies is equal to  $5 \times 10^{-5}$  for all paths and receiver elements.

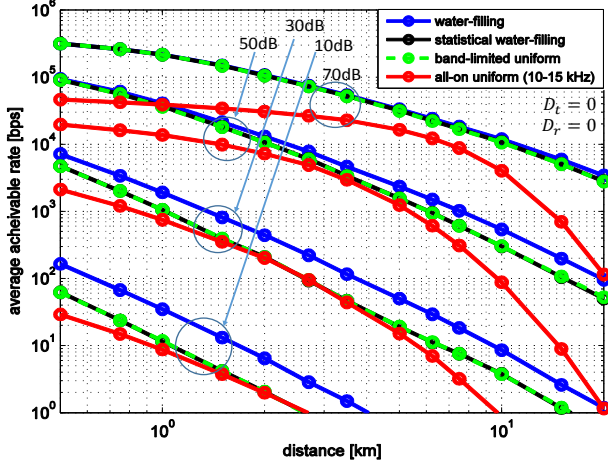


Fig. 3. Average rate vs. distance between the transmitter and the receiver for multiple values of transmit power (10, 30, 50, and 70 dB). Statistical channel parameters ( $A_k$  and  $\sigma_{z_k}$ ) are calculated as shown in Fig. 1. Feedback is assumed to be instantaneous.

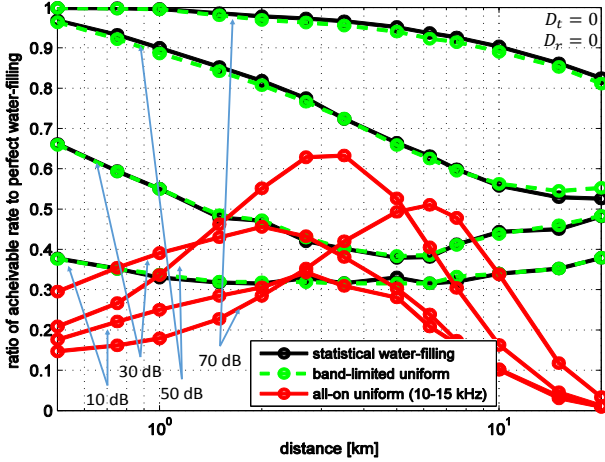


Fig. 4. Ratio of average rate for statistical water-filling, frequency band selection, and all-on uniform power allocation policies to water-filling vs. the distance between the transmitter and the receiver. System parameters are the same as in Fig. 3.

As the power increases (e.g. at 70 dB), however, all methods except the all-on uniform policy perform similarly.

Fig. 5 demonstrates how the benefits of water-filling turn into a loss with introduction of feedback delay even for SNR-starved channels. For example, assuming one step correlation coefficient  $\rho = 0.8$ ,  $\bar{K} = 2$ , and feedback delay as long as 20 OFDM blocks (which is equivalent to the round-trip propagation delay of a 1 km link), water-filling is outperformed by the other strategies even if the receiver knows the channel perfectly. The performance of all-on uniform policy depends on the preselected frequency band. For example, the frequency range between 10 kHz - 15 kHz is a good choice for a 3 km link if an achievable rate of no more than 10 kbps is desired, while it is far from optimal for higher transmit powers or longer distances.

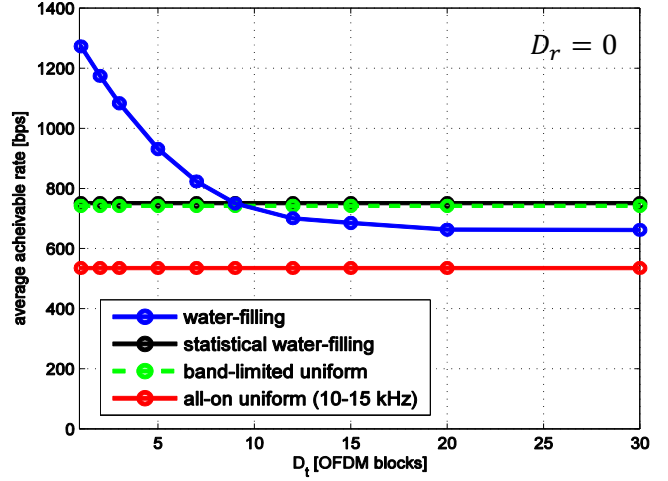


Fig. 5. Achievable rate vs. delay for multiple power allocation policies. Distance between the transmitter and the receiver is 1 km, normalized transmit power is 30 dB and  $\rho = 0.8$ . The rest of the parameters are the same as Fig. 3.

## V. CONCLUSION

We extended the results obtained in [6] which considered short term channel variations over a statistically flat channel with single element receiver in two ways: we considered long-term frequency-dependent channel statistics, and included multiple receiving elements. Numerical results, obtained via simulation and experimentally measured channels, indicate that the achievable rate increases almost linearly with the logarithm of the number of receiver elements. On comparing the four power allocation policies – water-filling, statistical water-filling, frequency band selection, and all-on uniform – the first three offer little improvement for high-SNR scenarios. These three power allocation policies will outperform the all-on uniform policy when long-term frequency dependent attenuation and noise are considered. Unlike the results in [6] which focuses on the multipath and channel estimation, this analysis assumes perfect channel estimation and studies the effect of feedback delay on power allocation strategies. In the presence of delay, water-filling will include the gross effects of frequency attenuation and frequency noise. However, it will not perform as well as statistical water-filling due to short-term variation of the channel [6], [7]. The simple frequency band selection policy which requires minimal feedback and uniform power allocation across the selected band emerges as a justified choice for underwater acoustic channel transmission. More information about this work can be found in [7]. Future work will address long-term channel variation, and the attendant methods for rate maximization via power control.

## ACKNOWLEDGMENTS

This work was supported by the following grants: ONR N00014-09-1-0700, NSF CNS-1212999.

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