Packet Scheduling for Underwater Acoustic Sensor Network Localization

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Abstract—This article considers the problem of packet scheduling for localization in an underwater acoustic sensor network where sensor nodes are distributed randomly in an operating area. Our goal is to minimize the localization time, and to do so we consider two packet transmission schemes, namely collision-free, and collision-tolerant. Through analytical results and numerical examples the performances of these schemes are shown to be comparable. In general, for small packet length (as is the case for a localization packet) and large operating area (above 3km in at least one dimension), the performances of the collision-tolerant protocol is superior to its collision-free counterpart. At the same time, the anchors work independently of each other, and this feature simplifies the implementation process.

I. INTRODUCTION

The data packets in an underwater acoustic sensor networks (UASN) are usually meaningless if they are not tagged with the time and the location of their origin. In this sense, localization is an indispensable task in different applications such as tsunami monitoring, oil field inspection, shoreline surveillance, and so on. Due to the challenges of underwater acoustic communications such as low data-rate, and long propagation delays with variable sound speed [1], a variety of localization algorithms have been introduced and analyzed in the literature which are relatively different from the ones studied for terrestrial wireless sensor networks [2] [3].

For an underwater sensor node to determine its location, it usually measures the time of flight to several anchors with known positions, estimates its distance to them, and then performs multilateration. Other approaches may be employed for self-localization, such as finger-printing or angle of arrival estimation. Nevertheless, packet transmissions from anchors are required in all these approaches.

Although a great deal of research exists on underwater localization algorithms, little has been done on determining how the anchors should transmit their packets to the sensor nodes. In long base-line (LBL) systems where transponders are fixed on the sea floor, an underwater node interrogates the transponders for round-trip delay estimation [4]. In the underwater positioning scheme of [5], a master anchor initiates a beacon signal periodically, and other anchors transmit their packets in a given order after the reception of the beacon

from the previous anchor. In reactive localization [6], an underwater node initiates the process by transmitting a "hello" message to the anchors in its vicinity, and those anchors that have received the message correctly transmit their packets. An existing medium access control (MAC) protocol may be used for packet exchanging; however, there is no guarantee that it will perform satisfactorily for the localization task. The performance of localization under different MAC protocols is evaluated in [7], and it is shown that a simple carrier sense multiple access protocol performs better than recently introduced underwater MAC protocols such as T-Lohi [8].

In our previous work, we considered optimal collision-free packet scheduling in a UASN for localization in single-channel [9] and multi-channel [10] scenarios. There, the position information of the anchors is used to minimize the localization time. In spite of the remarkable performance over other algorithms (or MAC protocols), these algorithms are highly demanding. Their main drawback is that they require a fusion center which gathers all the position information of the anchors, and decides for the time of packet transmission from each anchor. In addition, they need the anchors to be synchronized and equipped with radio modems in order to exchange information fast. In contrast, in this paper we consider packet scheduling algorithms that do not need a fusion center or synchronized anchors.

We assume a single-hop UASN where anchors are equipped with half-duplex acoustic modems, and can broadcast their packets based on two classes of scheduling: collision-free, where the probability that the transmitted packets collide with each other at the receiver of each underwater node is zero, or collision-tolerant, where the collision probability is controlled in such a way that each node can receive sufficient error-free packets for self-localization.

The structure of the paper is as follows. Section II describes the system model, and explains how self-localization can be implemented. The problem of minimizing the localization time in the collision-free and collision-tolerant packet transmission is formulated and analyzed in Section III and Section IV, respectively. It is also shown how the minimum localization time can be obtained for each approach. Section V compares the two classes of localization packet scheduling through several numerical examples. Finally, we conclude the paper in Section VI, and mention some future works.

II. System model

We consider a UASN consisting of M underwater nodes and N anchors as shown in Fig. 1. Each anchor in the network encapsulates information about its ID, its location and time of transmission into a localization packet, which is broadcast to the network based on a given protocol, e.g., periodically or upon the reception of a request packet from a sensor node [11]. The following assumptions hold in the network model.

- The anchors are roughly synchronized with each other; however, the sensor nodes may not be synchronized with the anchors. This is a reasonable assumption because anchors are usually located on the surface and can be equipped with a GPS. It should be noted that no synchronization is needed when anchors use an on-demand packet transmission protocol, i.e., when an underwater node initiates the localization protocol, and the anchors are notified after reception of the transmitted packet.
- Anchors and sensor nodes are equipped with half-duplex acoustic modems, meaning they cannot transmit and receive simultaneously.
- Sensor nodes are located randomly in an operating area according to some probability density function (pdf). We assume that the distance d between a sensor node and an anchor is distributed according to the pdf $g_D(d)$. It is further assumed that the pdf of the distance between anchors is $f_D(d)$. The pdfs can be estimated from the empirical data gathered during past network operations.
- Although the concept of this article can be extended to a multi-hop network, in this work we consider a single-hop network where all the nodes are within the communication range of each other. In addition, it is assumed that in the absence of packet collision, the probability of the packet loss between an anchor and a sensor node is p_l.

Each sensor node can determine its location if it receives at least K localization packets from K different anchors. The value of K depends on the geometry (2D or 3D), sensor nodes being synchronous or asynchronous, and other factors such as whether sound speed estimation is required or not. In general, the value of K is usually 3 for a 2D operating environment and 4 for a 3D if synchronous localization algorithms are employed. In a situation where the underwater nodes are equipped with pressure sensors, three different successful packets would
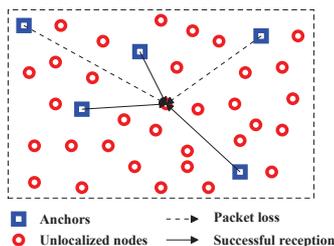
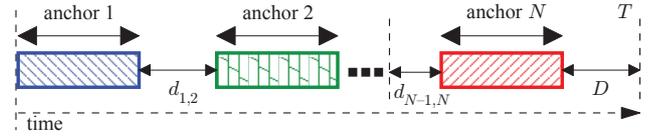Fig. 1: Anchors and sensor nodes are uniformly distributed in a rectangular area.

Fig. 2: Packet transmission from anchors in a collision-free scheme. Here, each anchor transmit its packets according to its index value (ID number).

be enough in a 3D synchronous localization algorithm [12]. The time it takes for an underwater node to gather at least K correct packets from K different anchors is called the localization time, t. A shorter localization time allows for a more dynamic network and reduced energy consumption. In the next section, we formally define the localization time, and show how it can be minimized for the collision-free and collision-tolerant packet transmission protocols.

III. Collision-free packet scheduling

Collision-free localization packet transmission is analyzed in [9], where it is shown that in a fully-connected (single-hop) network, based on a given sequence of the anchors' indices, each anchor has to transmit immediately after receiving the previous anchor's packet. Furthermore, it is shown that there exists an optimal ordering sequence which minimizes the localization time; however, to obtain that sequence a fusion center and the positions of the anchors are required. In a situation where this information is not available, we may assume that anchors simply transmit in order of their ID numbers as illustrated in Fig. 2.

Under this condition and with the assumption of no packet loss between the anchors, the localization time can be obtained as

$$t = NT_p + \frac{1}{c} \sum_{j=1}^{N-1} d_{j,j+1} + \frac{D}{c}, \tag{1}$$

where T_p is the packet length, $d_{j,j+1}$ is the distance between the j-th and $(j+1)$-th anchors, c is the propagation speed, and D is the distance corresponding to the maximally separated sensor-anchor pair. The maximum propagation delay, $\frac{D}{c}$, has been added to t in order to ensure that the last transmitted packet would reach the farthest node. Note that in (1) no packet-loss between the anchors is considered. However, if an anchor does not receive the previously transmitted packet (e.g., due to packet loss), it waits for a predefined duration from the starting time of localization, and then transmits its packet, similarly as introduced in [13]. With slight modification of [13], the waiting time for the j-th anchor could be as little as $t_k + (j-k)\left(T_p + \frac{D}{c}\right)$, where k is the index of the anchor whose packet is the last one which has been received by the j-th anchor, and t_k is the time at which this packet was received by the j-th anchor (from the starting time of localization). In the worst case when all the packets between anchors are lost, the localization time would converge to

$$t_{\text{upp}} = N\left(T_p + \frac{D}{c}\right), \tag{2}$$

which is an upper bound on the localization time, and is

equivalent to the packet transmission based on time division multiple access (TDMA) with time-slot duration $T_p + \frac{D}{c}$. In contrast, the result of (1) gives us a lower bound on the time that is required to perform collision-free localization.

The lower bound on the collision-free localization time (1) is a random variable and its pdf, $f_T(t)$, depends on the pdf of $\bar{d} = \sum_{j=1}^{N-1} d_{j,j+1}$. The pdf of $\bar{d}$ can be calculated as

$$f_{\bar{D}}(z) = \underbrace{f_D(z) * f_D(z) * \ldots * f_D(z)}_{N-1 \text{ times}}. \quad (3)$$

Given the pdf of the collision-free localization time, $f_T(t)$, the minimum time (including maximum propagation delay) it takes for the anchors to transmit their N packets with probability P_{tt} gives us minimum collision-free localization time which can be obtained by solving

$$P_{tt} = \int_{t=0}^{t_{\min}} f_T(t)dt. \quad (4)$$

Another system requirement is the probability with which a node can localize itself. If this probability is required to be above a design value P_{ss}, the necessary number of anchors is determined as the smallest N for which

$$\sum_{k=K}^{N} \binom{N}{k} (p_g)^k (1 - p_g)^{N-k} \geq P_{ss} \quad (5)$$

where p_g is the probability that a transmitted packet reaches a sensor node successfully and can be calculated as

$$p_g = (1 - p_l) \int_{bN_0B}^{\infty} f_{X_0}(x)dx, \quad (6)$$

where N_0B is the noise power, b is the minimum SNR at which a received packet can be detected at the receiver, and $f_{X_0}(x)$ is the pdf of the received signal power which will be derived later in the next section.

It is worth mentioning that instead of increasing the number of anchors, in a mobile scenario, one can repeat packet transmissions from K anchors multiple times to add diversity to the packet detection. That would change (1) and the pdf of the localization time to some extent; however, this approach is not analyzed in this paper.

IV. COLLISION-TOLERANT PACKET SCHEDULING

To avoid the need for coordination among anchor nodes, in a collision-tolerant packet scheduling, anchors work independently of each other. During a localization period or upon receiving a request from an underwater node, they transmit randomly, e.g. according to a Poisson distribution with an average transmission rate of λ packets per second. Packets transmitted from different anchors may now collide at a sensor node, and the question arises as to what is the probability of a successful reception. This problem is a mirror image of the one investigated in [14] where sensor nodes transmit their packets to a common fusion center. Unlike [14], where the sensors know their location, and power control fully compensates for the known path-loss, path-loss is not known in the present scenario, and represents the most dominant factor, while fading

and shadowing are absorbed into the probability packet loss. In this regard, the signal received at the m-th sensor node from the n-th anchor is

$$v_{m,n}(t) = c_{m,n}v_n(t) + i_m(t) + w_m(t), \quad (7)$$

where $v_n(t)$ is the signal transmitted from the n-th anchor, $c_{m,n}$ is the channel gain, $w_m(t)$ is the additive white Gaussian noise with power N_0B, and $i_m(t)$ is the interference caused by other anchors whose packets overlap with the desired packet,

$$i_m(t) = \sum_{k \neq n} c_{m,k}v_k(t - \tau_k), \quad (8)$$

where τ_k is the difference in the arrival times of the interfering signals w.r.t. the desired signal, and is an exponentially distributed random variable. The signal to noise ratio (SNR) at the receiver depends on the interference level, and can be calculated as

$$\gamma_{m,n} = \frac{X_0}{I_0 + N_0B}, \quad (9)$$

where $X_0 = |c_{m,n}|^2 P_0$ is the power of the signal of interest, P_0 is the anchor's transmit power, and I_0 is the total interference power which can be expressed as

$$I_0 = \sum_{i=1}^{q} |c_{m,k_i}|^2 P_0 \quad (10)$$

where q is the number of interferers, and k_i is the index of the i-th interferer. Moreover, with a simple path-loss attenuation model we can formulate the attenuation of the signal power as

$$|c_{m,n}|^2 = \alpha_0 \left(d_0/d_{m,n}\right)^{n_0}, \quad (11)$$

where α_0 is a constant, d_0 is the reference distance, and n_0 is the path-loss exponent. Using (11), the pdf of the received signal power of the desired signal is

$$f_{X_0}(x) = \frac{d_0}{n_0} \left(P_0\alpha_0\right)^{\frac{1}{n_0}} \left(\frac{1}{x}\right)^{\frac{1}{n_0}+1} g_d\left(d_0(\frac{P_0\alpha_0}{x})^{\frac{1}{n_0}}\right), \quad (12)$$

and the pdf of the interference can be obtained as

$$f_{I_0}(z) = \underbrace{f_{X_0}(z) * f_{X_0}(z) * \ldots * f_{X_0}(z)}_{q \text{ times}}. \quad (13)$$

The probability that a packet is received correctly by an underwater node then is [14]

$$p_s = (1 - p_l) \sum_{q=0}^{N-1} P(q)p_{s|q}, \quad (14)$$

where $P(q) = \frac{(2N\lambda T_p)^q}{q!}e^{-2N\lambda T_p}$ is the probability that q packets interfere with the desired packet, and $p_{s|q}$ is the probability that the desired packet survives under this condition which can be obtained as

$$p_{s|q} =$$
$$\begin{cases} \int_{bN_0B}^{\infty} f_{X_0}(x)dx & q = 0 \\ \int_b^{\infty} \int_{N_0B}^{\infty} f_{X_0}(\gamma w)f_I(w - N_0B)w dw d\gamma & q \geq 1 \end{cases} \quad (15)$$

where $w = I_0 + N_0 B$.

In addition, it should be noted that redundant successfully received packets from an anchor are not useful for localization, and will therefore be considered as one correctly received packet. However, they may be used to reduce the effects of noise on the range estimation, or in mobile scenarios where the anchors are moving they can be used for range tracking [15]. However, we will not consider that in this paper.

The probability of receiving a useful packet from an anchor during t seconds can now be approximated by [14]

$$p_g = 1 - e^{-p_s \lambda t}, \qquad (16)$$

and eventually the probability that an underwater node can accomplish self-localization during t seconds using N anchors can be obtained as

$$P_{ss} = \sum_{k=K}^{N} \binom{N}{k} p_g^k (1 - p_g)^{N-k}, \qquad (17)$$

which is equivalent to the probability that a node can receive at least K different localization packets during t seconds.

Given the number of anchors N, and a desired probability of successful self-localization P_{ss}, one can determine p_g from (17), and λ and the minimum localization time jointly from (14) and (16). Similarly to the collision-free scheme, we then add the maximum propagation delay to the minimum t that is obtained from (14) and (16). This value is then considered as the minimum localization time $t_{\min}$, for the collision-tolerant scheme.

V. NUMERICAL RESULTS

For the numerical results, a two-dimensional rectangular-shape operating area with length D_x and width D_y is considered with uniformly distributed anchors and sensors. There is no difference in how the anchors and sensor nodes are distributed in the environment, and therefore we have $f_D(d) = g_D(d)$ which can be obtained as (see Appendix I)

$$f_D(d) = \qquad (18)$$
$$\frac{2d}{D_x^2 D_y^2} \left[d^2(\sin^2\theta_e - \sin^2\theta_s) + 2D_x D_y(\theta_e - \theta_s) \right.$$
$$\left. + 2D_x d(\cos\theta_e - \cos\theta_s) - 2D_y d(\sin\theta_e - \sin\theta_s) \right]$$

where θ_s and θ_e are related to d as given in Table I.

TABLE I: Values of θ_s and θ_e based on distance d.

distance	θ_s	θ_e
$0 \le d \le D_y$	0	$\frac{\pi}{2}$
$D_y \le d \le D_x$	0	$\sin^{-1}\frac{D_y}{d}$
$D_y \le d \le \sqrt{D_x^2 + D_y^2}$	$\cos^{-1}\frac{D_x}{d}$	$\sin^{-1}\frac{D_y}{d}$

The parameter values for the numerical results are listed in Tables II, and III, and for the all numerical results we use these values unless otherwise stated. The transmission power is set in such a way that for any distance, in the collision-free scheme, the SNR is greater than b. In the other words, the

TABLE II: System parameters

Name	D_x	D_y	P_0	α_0	d_0	n_0	$N_0 B$
Value	$3c$	$3c$	1	1	1	2	-82 dB

probability of packet detection when there is no collision and no packet-loss is 1.

TABLE III: Design parameters. Note that, N depends on the values of the other parameters (p_l, and P_{ss}), and is set in such a way to agree with them.

Name	T_p	b	p_l	P_{ss}	P_{tt}	K	N
Value	0.1	4	0.1	0.99	0.99	4	7

The packet length depends on the system bandwidth. In an underwater acoustic communication system the useful bandwidth in turn depends on the transmission distance. For short distances the useful bandwidth (or the maximum achievable data rate) is high, while for long distances it is lower [16]. The value of T_p is selected to be 100ms which is sufficient to carry a localization packet (for instance @2kpbs, 80 bits may be reserved for useful information, and 60ms as guard time, training sequence and error correcting code).

Fig. 3 shows the probability of successful self-localization in the collision-tolerant scheme as a function of λ and the corresponding value for t. It can be observed that there is an optimal value of λ (denoted by λ_{opt}) which corresponds to the minimal value of t ($t_{\min}$). Furthermore, for the values of t greater than $t_{\min}$, a range of values for $\lambda \in [\lambda_{\text{low}}, \lambda_{\text{upp}}]$ can attain the probability of self-localization. In this case, the lowest value for λ should be selected to minimize the transmission energy consumption.

Fig. 4 shows the probability of a correct packet reception versus the number of interferers (the effect of packet-loss is not included in the figure, and the desired P_{ss} is set to 0.90 in this example). As it was mentioned before, when there is no interference, the probability of packet reception is 1. Yet, when there is an interferer, the chance of correct reception of a packet becomes small (0.11 for this example), and as the number of interferers grows, it gets smaller. The probability that two or more packets overlap with each other is also depicted in this figure for the three values of λ shown in Fig. 3. It can be seen that as the value of λ is reduced from λ_{opt} (which is equivalent to a larger t), the probability of collision
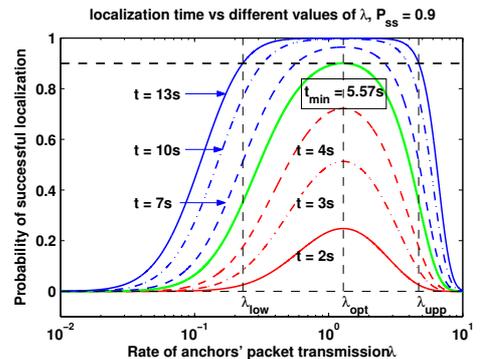

Fig. 3: Probability of successful localization for different values of λ and t.
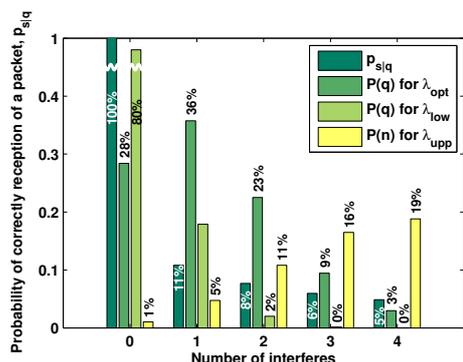
Fig. 4: Probability of successful packet reception versus different numbers of interferers, and the probability that these interferers collide with the desired packet. For this figure λ_{low}, λ_{opt} and λ_{upp} are chosen from Fig. 3.

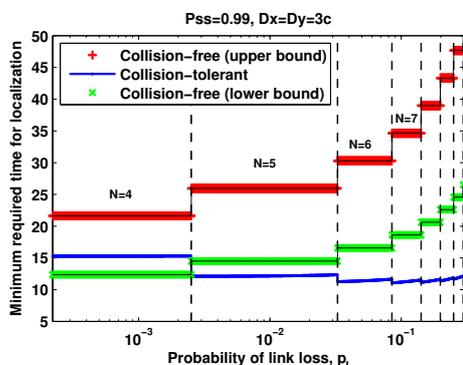Fig. 6: Effect of packet length on the minimum required time fi localization.

Fig. 5: Effect of link-loss probability on the minimum required time for localization. The greater the value of p_l is, the more number of anchors is required in collision-free protocols.

Fig. 7: Effect of the operating area size on the required localization time.

gets smaller. This increases the chance of a correct packet reception, and reduces the energy consumption in packet transmission by an anchor.

Link-loss is a phenomenon that is common in underwater acoustic systems because of many reasons such as location-dependent fading, shadowing, noise, and so on. Fig. 5 shows the minimum required time for localization versus the probability of link-loss. As p_l increases, more anchors are required for a collision-free localization. In this figure, for a given p_l, the number of anchors N, is calculated using (5), which is then used to calculate t_{min} for the collision-free and collision-tolerant cases. Each increase in t_{min} for the collision-free schemes indicates that the number of anchors has been increased by one. We also note that for a given number of anchors, the performance of the collision-free algorithm is constant over a range of p_l, but that of the collision-tolerant increases slightly as p_l gets larger in that region. However, the collision-tolerant approach performs better for a wide range of p_l, and can be implemented in practice with low computational complexity since the anchors work independently of each other.

As it was mentioned before, the localization packet is usually short, and carries information about the time of transmission and location of the anchor. In an asynchronous localization it may also include the time that the anchor
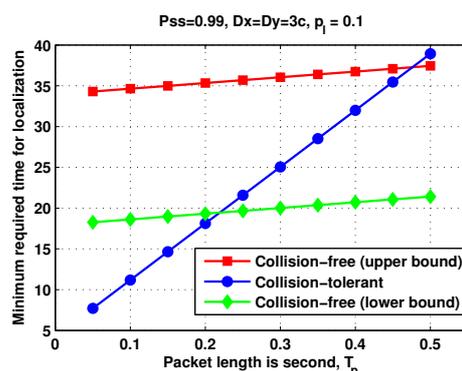
receives a request packet from an underwater node. As it is shown in Fig. 6, the length of the localization packet plays a significant role in the collision-tolerant algorithm. The minimum localization time grows almost linearly w.r.t. T_p in all cases; however the rate of growth is much higher for the collision-tolerant system than for its collision-free counterpart. At the same time, as shown in Fig. 7, the size of the operating area has a major influence on the performance of the collision-free system, while that of the collision-tolerant system does not change very much. It can be deduced that in a network where the ratio of the packet length to the maximum propagation delay is low, the collision-tolerant algorithm outperforms the collision-free one.

VI. CONCLUSIONS

We have considered two classes of packet scheduling for self-localization in an underwater acoustic sensor network, one based on a collision-free design and another based on a collision-tolerant design. In collision-free packet scheduling, the time of the packet transmission from each anchor is set in such a way that none of the sensor nodes experiences a collision. In contrast, collision-tolerant algorithms are designed so as to control the probability of collision to ensure successful localization with pre-specified reliability. The performance of the two classes of algorithms was shown to be comparable. Moreover, when the ratio of the packet length to the maximum propagation delay is very low, the collision-tolerant protocols require less time for localization in comparison with the

collision-free ones for the same probability of successful localization. Furthermore, in the collision-tolerant approach there is no order in the anchors' packet transmissions, and they work independently of each other. As a result, there is no need for a fusion center, and the anchors do not need to be synchronized. These features make the collision-tolerant localization scheme appealing for a practical implementation. In the future, we will analyze the localization accuracy under the collision-tolerant packet transmission scheme, and extend this work to a multi-hop network.

APPENDIX I: DISTRIBUTION OF THE MUTUAL DISTANCE

In this appendix, we derive the pdf of the distance between two nodes located uniformly random in a rectangular region as shown in Fig. 8. Under this condition the pdfs of x and y
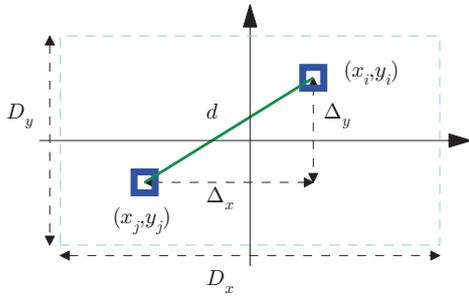

Fig. 8: Two nodes randomly located nodes in a rectangular operating area.

projections are

$$f_{\Delta_X}(\Delta_x) = \frac{2}{D_x^2}(D_x - \Delta_x), \ 0 \le \Delta_x \le D_x \quad (19a)$$

$$f_{\Delta_Y}(\Delta_y) = \frac{2}{D_y^2}(D_y - \Delta_y), \ 0 \le \Delta_y \le D_y, \quad (19b)$$

and since they are independent, the joint pdf in a polar coordinates (see Fig. 9) is

$$f_{D,\Theta}(d,\theta) = \frac{4d}{D_x^2 D_y^2}(D_x - d\cos\theta)(D_y - d\sin\theta). \quad (20)$$

By taking an integral over θ, the pdf of the distance follows (18). This pdf is shown in Fig. 10.
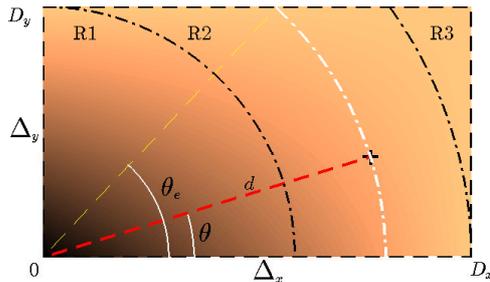

Fig. 9: Illustration of the parameters and their relations to each other in calculating the probability density function of the distance between two nodes located uniformly at random.

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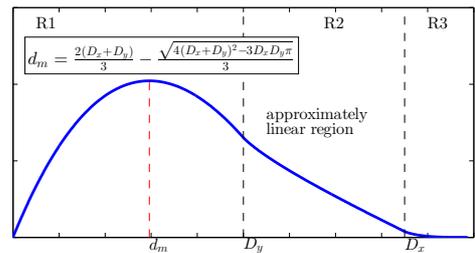
Fig. 10: Probability density function of the distance between two uniformly randomly located nodes. d_m is the point at which the maximum of the pdf occurs.